tracery yet, it gave us enough confidence so that we did fix the lower chord, and its three-arched truss envelope as the final shape of the one we were going to keep.

We knew that its interior tracery was not beautiful yet, and I knew it was far from acceptable for the project — especially judging it on the mirror-of-the-self criterion. However, by this stage, we had in principle consolidated our decision to try and make a truss with beautiful geometrical tracery, using curved members, and not obviously based on simple truss design. From this point of view, the free form of the first curved tracery was something to start with, even though it was not beautiful. As it turned out, it was not well-behaved structurally either.

9 / GOING ON WITH THE UNFOLDING PROCESS FOR THE TRUSS: FINITE ELEMENT ANALYSIS

6. *First finite element model.*

Of course we had no idea whether this first tracery would be well-behaved structurally. We therefore made a first finite element model to find out how the forces went. We could immediately see some very bad behavior. The forces were several times over limits in several places, huge shears at the base, and moments too big in some of the curves.

7. *First scissors truss.*

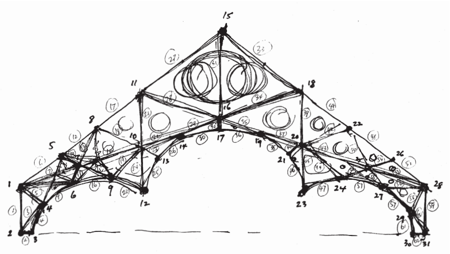
At this stage I decided to go back to structural behavior, and started by trying to define the most efficient truss which was consistent with the three arch profile we had chosen.

It seemed to me most likely that the arches could be made to work by placing a tension member in the position typical in a classic scissors truss. I sketched this out in rough, and one of our apprentices built a miniature one in concrete, one inch thick, with a span of 3 feet. It was not appealing. The apparent simplicity of the structural lines, when given width, made a mishmash of shapes which was inconsistent, geometrically, with the beauty of the three-arch form. (Two minor points about its ugliness: The sharp points where the arches meet, and its lack of three-dimensional relief — like a slab of chocolate. These problems were solved later.)

I then decided, myself, to test a series of computer models, in rapid succession, and started an intensive ten-day session of uninterrupted computer work, to try and find a solution which was both beautiful and structurally efficient.

8. *Second scissors truss.*

In spite of the ugliness of the first scissors truss, I did not give it up right away. Before going ahead, I still wanted to find out what kind of truss would be structurally efficient for the three-arch envelope we had chosen for the truss. Since the first scissors truss was ugly geometrically, I did not even take time to test it in the computer, and instead decided to try sketching a second scissors truss, and ran a finite-element model to determine its behavior.



8. Second scissors truss sketch and its nodes

I drew what seemed like a perfect triangulation of the curvilinear shape above the three-arch bottom chord (see drawing above). In this drawing, I tried to make all the triangles as neat and similar as possible with good angles, and each triangle being as near a 60–60–60 triangle as possible, in a way that was consistent with the overall geometry.

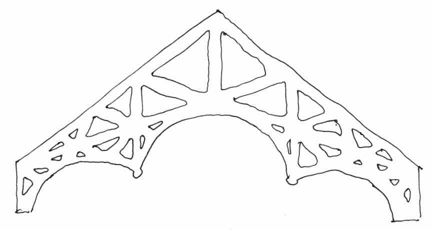
Since this model was only for finite element analysis, I kept it as a pure structure of nodes and thin straight lines representing members.

This truss had beautiful structural behavior. Bending moments were low, shears were low. Everything was acting well within its capacity.

9. *Thick version of scissors truss.*

From a structural point of view, Gary and I were both pleased with the behavior of the efficient truss. The line drawing of the truss looked very nice, so we thought it had excellent promise. It also looked as if there were a good cooperation between its geometric features and its structural features.

When you use nicely shaped triangles, the triangles naturally get smaller where the area of the truss gets tighter over the side arches. This has the feature that where the triangles get smaller towards the bottom, the shorter members and tighter mesh correspond to the zone with the biggest flow of forces. The arch members thus force a geometry which helps the structure in a natural way. Intuitively, it looked very promising.

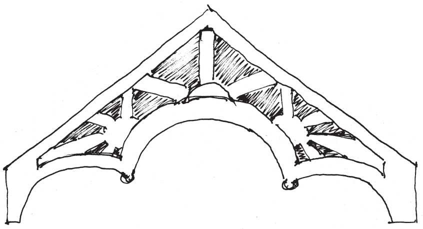


9. Thick version of scissors truss design

But the truss had a very disappointing problem. When I started giving the members real dimension in the computer model, the spindly beauty of the theoretical truss turned into an ugly and squat kind of shape. The truss was beautiful when its members were very thin, but became awkward and ugly when you made each of the members 6 or 9 inches wide as they would be in reality.

10. *Rising-sun truss.*

To improve the thick-scissors truss I tried to intensify the centers. This began to make something that had form and being, but it certainly still wasn't beautiful. On the mirror of the self test it was also still poor, indeed so far from the mark that I didn't even run a finite element analysis of it. It didn't seem worth the trouble.



10. Rising-sun truss

11. *Re-evaluation of behavior.*

At this stage Gary and I spent some time trying to understand the good behavior of the scissors truss in more detail. We read the output from the finite element analysis very carefully, poring over the tension, compression, bending and shear in every member.

One of the most surprising results of this work, was that we had apparently quite misunderstood the action of the so-called scissors truss. I had assumed that the key feature of the scissors truss was the tension force going in a long diagonal line tangent to the arches. With the three-arch form there could be no tension chord going across the bottom (as in a normal truss) but, somehow, the tension needed to resist the spreading of the roof still had to get taken care of. I had assumed that the scissors tension member tangent to the arches, was the most obvious way to do it. Of course, on this theory this tension member had to be straight line, and I assumed that it had been the lack of this straight line which had made the first curved truss behave badly.

However, when we looked carefully at the distribution of forces in the efficient truss (which included this straight line) it turned out that there was almost no tension being carried along this line! We had completely misunderstood the reason why the good truss was well-behaved.

What was actually happening was this: The bottom chord (the three-arched line) was acting in tension and bending. The top chord was acting in compression and bending. The members in between were alternating in compression and tension. In short, the truss was acting very much like a classic triangulated truss with tension in the bottom chord, but with the additional feature of bending in the bottom chord members to allow the tension to go round the curves. The major tension was not going above the arches and tangent to them, even though the straight line in the members gave an opportunity for it. It was going around the arch itself.

At this stage, it seemed as though the original idea of a floral truss ought to work after all. What we now knew, that we hadn't known before, was that the top and bottom members were doing most of the work anyway.

12. *Thick and thin members in the truss.*

Our understanding of the fact that the main flow of forces is in the top and bottom, coincided with a physical problem which had been bothering us for some time. Ever since the first concrete model, the flatness of the truss had been bothering me; we had been talking about needing two levels of thickness to create three-dimensional modelling in the surface of the truss.

We decided that from now on, we would use thick members for top and bottom, and thinner members for the struts between. As a result, the hierarchy of scales in the truss became more beautiful, and the truss became more personal feeling more "mirror-of-the-self"-like.

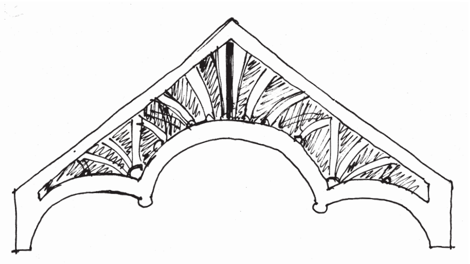
13. *Knobs at the meeting of the arches.*

One of the defects visible in the earliest truss drawings was the pair of strange-looking sharp points at the spots where the half-arches met the main arch. These points made little sense structurally since they created a potential danger of stress concentrations.

We may also see the same problem in deeper geometric terms by looking at the centers which the arches create. In order to intensify the centers formed by the arches, the points where the two arches meet must also become centers. To do this in one of my earlier sketches there had been a pair of sort of lamp-like ornaments hanging from these two points. I now introduced two circular knobs of concrete at the meeting point of the arches, and these became a permanent part of the truss design. In the following sketches, we see various forms of these knobs, and see how one in particular has the greatest ability to be a picture of the self.

14. *First plant-like truss.*

In line with this new understanding we had gained, I now began drawing a truss in which the top and bottom chords played the main role, and in which the in-between stuff was acting as a kind of stitching, just to hang them together.

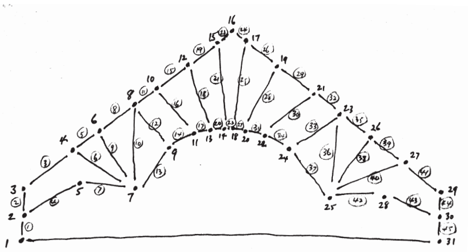


14. My first sketch of the plant-like floral truss

With the amount of understanding of the forces that I had from the earlier finite-element models, I now drew the nearest thing I could to a perfect system of centers that described the forces correctly. The two levels of thickness are visible in the drawing. This drawing, with its system of radiating curves, seems, for the first time, truly to *enhance* the wholeness of the three-arched truss shape.

15. *Finite-element model of the first plant-like truss.*

In the next model I introduced different member sizes into the finite element model, since the



15. Finite element model of plant-like floral truss

differentiations of size had, by now, become an important part of the feeling of the design. Even the relative thicknesses of the top chord (4 inches by 9.5 inches) and the bottom chord (4 inches by 15 inches) compared with the inner members (averaging 3 inches by 5.5 inches) was significant.

These members have roughly the following capacities:

Top chord (4 by 9.5) has 152,000 inch lbs. max bending, and 3,344 lbs. max shear.

Bottom chord (4 by 15) has 346,000 inch lbs. max bending, and 5,280 lbs. max shear.

Middle struts (3 by 5.5) have 47,000 inch lbs. max bending, and 1,452 lbs. max shear.

The finite element analysis showed generally good behavior, well within these limits, except for three problem areas. (a). A shear force of 7,400 lbs. in the top chord members near the peak (twice what the section could bear). (b). High shears in the arch where the tension comes in from above (4–8,000 lbs. — more than twice what the section could bear). (c). High shear in the vertical edge member caused by tension and compression twisting it (8000 lbs. — twice what that member could bear).

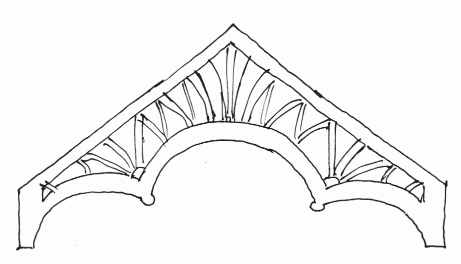
16. *Keeping the split stems in the middle.*

The 7,400 lbs. shear near the peak was caused by the fact that the pair of split tension members fails to resolve the compression at the peak. It would have been easy to solve this problem by making a single central tension strut as is typical in most trusses.

However, this change would have destroyed the beauty of the truss. While I was working on it I happened to discuss it with my students, and told them that I would refuse to make this change, even though it seemed obvious, because it would make the field of centers worse. I had an instinct, somehow, that the design of the truss, as drawn, with the split middle had a field of centers — and a resultant structural meaning — more subtle than that revealed by the finite element analysis. As it turned out later, this hunch was right.

17. *Trying more members.*

To get rid of the three shear problems I decided to try making the truss more like an ordinary truss by placing additional triangulated members to complete the rectangles. This change did slightly improve the behavior at the edge, but shear problems at the peak and main arch remained. In any case, once again, the subtle beauty and harmony of the truss got worse, not better, so we decided not to make these changes.



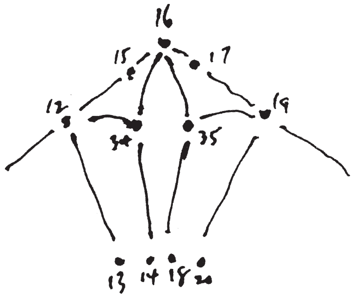
17. Trying more members

18. *Reducing shear at the apex.*

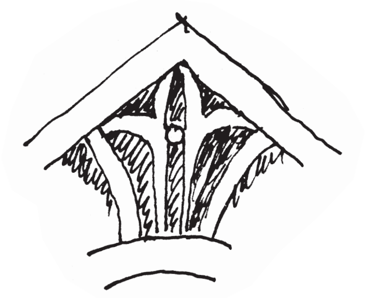
In order to reduce the shear at the top, I tried tying back the peak to the two tension struts, with a pure tension member. This didn't work. Most of the tension still went the other route, leaving shears almost as high as before. To solve it, we tried tying them back with another triangle, making a beautiful lily-like shape in the tracery near the peak.

This worked. The shear went down to 5,000 lbs. But now the bending moment in the peak became very high indeed, 114,000 inch lbs. To improve the situation we inserted another member going across horizontally, thinking this might work as a minor tension chord to hold the peak together, and so reduce bending at the

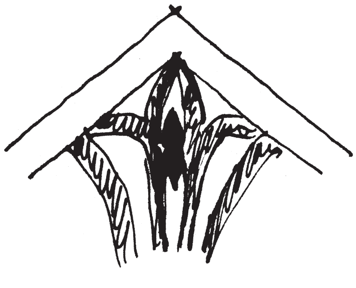
18a. Lily configuration



18b. Finite element model of the lily form



18c. Lily with a dot between the two vertical halves

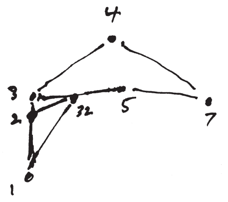


18d. The final version

peak. Oddly enough, it did work, but for quite different reasons. Under analysis it turned out to be in compression, which prevented creation of internal torques through the curved tension member and reduced bending and shear through the upper portion of the truss. To make the compression member intensify the field of centers, instead of spoiling it, I had to make it beautiful inside the lily. I tried making it like a shield shape or a dot. In the end a diamond-shaped piece turned to be best of all. It leaves the lily structure unspoiled.

19. *Reducing shear at the bearing points.* We made a similar series of changes to reduce shear on the side members. In this case, the crux of the problem was to lead the tension in the arch directly to node #3 so as to resolve the compression coming down the top chord. As we see in the following drawing, this change improves the field of centers in the geometry.

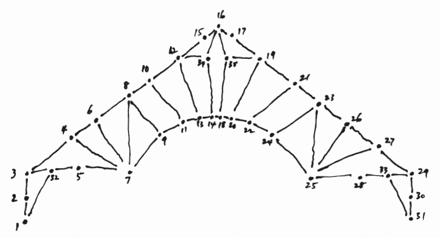
19a. Reducing shears at the bearing points



19b. Reducing shears at the bearing points

20. *Relaxing the tension arch.* Finally, at this stage we made a major breakthrough, almost by accident. In playing with various minor changes, we had begun to notice that the behavior of the truss was very susceptible to rather minor changes in the relative stiffness of different members.

During discussion of this point, I became very worried about the interpretation of the finite element model. In the model we had been assuming these various members were concrete, but in reality, a given concrete member, when in tension, will only realize the tension of its steel reinforcing bars, not of the concrete. This means that the stiffness which the members really have



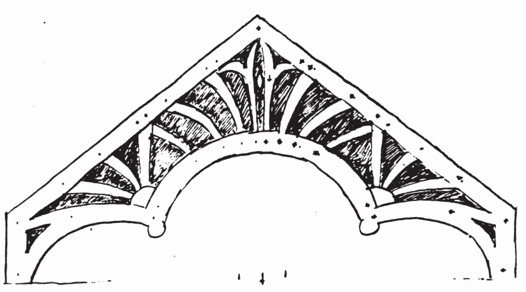
20. The tension arch

will be that defined by the configuration of the reinforcing bars, not that defined by the concrete. But the stiffness of the members considered as a configuration of rebars is often less than 20% of the stiffness of the full concrete section. There was a possibility, then, that the overall behavior of the truss might change drastically for the worse if we made this replacement, or might even collapse entirely. To check this worrying point Gary and I decided to try a finite element analysis in which all tension members would be given their steel stiffness, not the concrete stiffness.

The results of this analysis were fascinating. Instead of getting worse the behavior became *better*. All the shears in the top of the main arch went down to within acceptable limits. Better still the bending moments in the top chords also went down. And finally we noticed one more thing. In this modified truss the forces in several struts changed from compression to tension.

We now realized that as a whole the modified truss was working in an entirely unexpected way. The arch and struts were all together, working as a kind of tension network which resists the spreading of the rafter beams.

This was a highly unusual design, previously unknown to either of us. The three-arch arrangement which originally arose in response to the spatial configuration of the dining hall had finally been resolved in a form where this particular shape draws its structural strength from a novel way of working through a tension network arch. The oddity of the original configuration, caused by centers in the dining hall, had become a virtue in an entirely new structural design.



My original sketch of the final truss design

10 / EACH STEP USES THE FUNDAMENTAL PROCESS TO UNFOLD AN EARLIER WHOLENESS

At first sight, the various steps we took in the design of the San Jose truss seem very different from one another. In one step, to get the spacing of the trusses, we were looking at the windows to see if they have a beautiful shape. In another step we were looking at the distribution of tension and compression in the truss itself. In another we were concentrating on the shear force at three critical spots. In another, we were looking only at the beauty of the centers in the truss, to make it beautiful. In another we were looking at the thickness of the truss from the point of view of the steel bars crossing each other. In another, we were trying to find out how to make the formwork give us an offset between the inner members and the big members.

However, every single one of these steps, when interpreted correctly, was a structure-preserving transformation. And indeed, each of these structure preserving transformations was