500 ways you can do. Millions of possible
only 1% of what you want.
10,000 you want.

Ending.
on Japanese house.

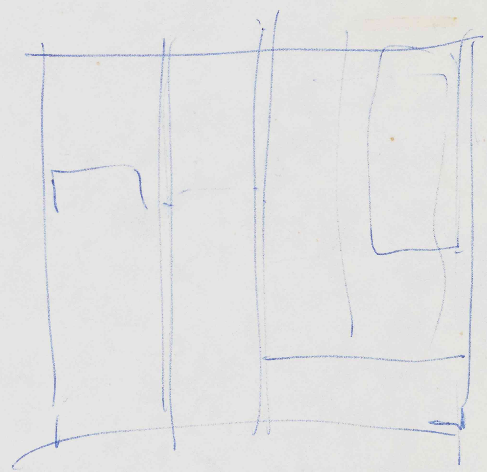
THE NON-MODULARITY THEOREM

Christopher Alexander
Artemis Anninou
Gary Black

1986

14.

6'10"



REVERSE.

Module creates ripples 100 times as large.

Window example.

INTRODUCTION

In the second half of the twentieth century, the American workplace has been dominated by configurations of mass-produced office furniture.

What we call furniture does not only include desks, filing cabinets, tables and chairs. It also includes panels, shelves, ceilings, and floors. All of it is modular. That means, that any one of these elements is made in a limited set of standard sizes. The various elements, in their various standard sizes, are then combined in various ways to produce a "layout".

The fundamental assumption underlying all furniture production in the 1970's and 80's, is this: Within reasonable limits, ANY desired layout can be made by combinations of the standard elements.

Of course it is widely recognised that the modularity of the elements may create constraints of a few inches here and there -- but the assumption is that these constraints have only very minor effects on the possible kinds of layout. It is assumed that they may slightly increase overall area, or that they may force certain dimensions an inch or two up or down from the ideal -- but it is assumed that beyond that, there is no harmful effect from the modularity of the elements, and no harmful impact on the configurations which can be produced with them.

In order to make this assumption quite precise, we must add to it, a definition of the modules in current use.

Available tables, for example, come in increments of 12 inches. You can buy a four foot, a five foot and a six foot table. In this case, then, we shall speak of a 12 inch module.

Available panel sizes from Haworth, come in a range of sizes which permit combinations of lengths that vary by increments of 8 inches. In this case, then, we shall speak of an 8 inch module.

The module we shall be referring to, throughout this article, is thus not the size of the element, but the size of increments which are available among the elements of a particular type. Typical modules in the present office furniture industry, range from about 4 inches to about 16 inches, and most of them are on the order of about 8 inches.

We may now sum up this general assumption, in the following proposition, which we shall call the modularity assumption:

ALL OFFICE CONFIGURATIONS WHICH ARE NECESSARY FOR HUMAN COMFORT AND EFFICIENT WORK CAN BE PRODUCED, TO WITHIN A REASONABLE AND ACCEPTABLE TOLERANCE, BY COMBINATIONS OF AVAILABLE MODULAR ELEMENTS.

This assumption is fundamental to the furniture industry as we know it. If this assumption were to be proven wrong, then the whole furniture industry would face massive reorganisation, which would profoundly shake the nature of the American workplace.

In this article we shall argue that the modularity assumption is indeed WRONG. We shall argue that, on the contrary, the modular nature of elements in a furniture system is extremely harmful, to an extent not even suspected up until now, and that comfortable and efficient workplaces ~~must be~~ are drastically harmed by the use of modular elements.

We shall argue that the modularity assumption quoted above, is a naive and erroneous assumption, which utterly fails to recognise certain deep interactions which occur in the nature of space, and that it is can be shown that a functionally adequate work enivironment can only be produced by elements whose module is several orders of magnitude smaller than the modules currently in use.

We shall argue, in short, that for purely mathematical reasons, which have only to do with the nature of space, and the nature of geometrical combinatorics, it simply is not possible to create adequate levels of comfort and efficiency, by combining crude furniture and partition modules in space.

We shall embody this statement in a loose theorem, which relates the grain of a configuration, and its level of functional order, to the size of module which can realise this necessary order of the configuration. As we shall see, the module necessary to achieve good results, is approximately one hundredth of the size of the key dimensions in the configurations desired.

The key dimensions in the human workplace are

Our modularity theorem then implies that the

~~In practical terms, this means that the~~ largest module which can produce comfort in the human workplace, ~~whose key dimensions are~~ on the order of three to ten feet (1 to 3 meters) is a module on the order of about 1/4 to 1/2 inch, or about one centimeter.

Presently furniture uses a module of about 8"

Our theorem then

means ~~implies~~

~~This single proposition, then shows~~ that, ~~in~~ all ~~any practical sense at all~~, modular mass produced furniture as currently produced by the present generation of American manufacturers, is doomed to produce disfunctional and harmful environments.

The definition of an alternative system of production, which is capable of making the fine adaptations necessary to guarantee GOOD configurations, ~~is~~ will be taken up in a separate article.

-oOo-

PART 1: MATHEMATICAL AND BIOLOGICAL EXAMPLES

In order to explain the general line of argument that will be followed in this paper, we shall begin with certain simple examples, that illustrate the general proposition that desirable configurations cannot necessarily be realised by combining modular elements.

We shall start with a series of examples which demonstrate that the kind of difficulty we are talking about is not "practical" or "technical" -- but mathematical. We are talking about things which are not just difficult, but really IMPOSSIBLE.

1. HEXAGONAL TILING OF THE SPHERE.

It is widely known that it is possible to tile an infinite plane, with hexagonal tiles.

HEXAGONAL TILES

One might assume, then, that by bending the tiles, and curving their edges a bit, it would also be possible to tile a sphere with hexagons.

However, this is is not difficult. IT IS IMPOSSIBLE. it is always necessary to insert a few pentagons. We can easily prove that there is no way whatsoever of doing it, by using Euler's theorem. (See Weyl, Symmetry, Princeton, ...).

WEYLS EXAMPLE OF TILING ON A SPHERE

Here we have one example of a case, where the exclusive use of a simple modular element, simply CANNOT produce a given desired configuration.

2. CRYSTAL DISLOCATIONS.

Consider, secondly, the configurations which can be made by infinite arrays of small ball bearings or bubbles. These arrays have been used as two dimensional analogues of crystal structure, to study large scale order in crystals.

DISLOCATIONS IN A RAFT OF SPHERES

Theoretically, the identical spheres can pack for ever in the plane, to form an infinite modular array.

In practice, however, what happens is quite different. The modular array goes for a certain distance, and there is then a rift, or crevasse, before the array starts up again. Often these rifts or dislocations separate parts of the array that are not aligned. Sometimes they are aligned, but the dislocation occurs anyway.

The dislocations occur almost universally in real crystals. They come about for various reasons. Some are caused by slight imperfections in the spheres, which "build up" and get more severe as the pattern crosses space. Others are caused by the fact that the infinite arrays start growing in more than one place, and then have no way of joining neatly, when the arrays meet.

Here we have a case, where identical modules cannot easily create even the simplest type of large-scale order. The large scale order forces a break in the array, simply as a result of the way things fit together.

What is remarkable, is that the ~~[illegible]~~ size of the rifts, and distortions caused in the large scale order, are about 100 times the size of the module.

Thus a tiny imperfection or aberration in the modules or in their connection, causes large scale distortions 100 times as large.

3. THE CHESSBOARD TILING PROBLEM.

Here is another example where a simple module cannot be arranged in a desired combination.

Consider a chessboard in which two opposite corners have been removed. It is an arrangement of 62 small squares, with two opposite corners taken out.

BOARD WITH 62 SQUARES

Suppose we now have 31 rectangles, like dominoes, each an exact double square, and try to find a way of arranging these 31 double-squares on the 62-square array.

If we try it, it is very hard to do. Somehow, the extra rectangle always ends up in the wrong place, and a single square gets left over somewhere where you dont want it. In any case, it seems hard to do.

Once again, the fact is, THAT IT CANNOT BE DONE AT ALL. This can easily be proved, by a simple mathematical argument (Martin Gardner or other).

But the important point is, that at first sight there is no obvious reason why the 31 double-squares should not be fitted together, somehow, to cover the 62 squares.

Apparently, there is some aspect of the global order, which escapes our notice, and yet makes the task impossible. There are, apparently, s~~d~~ome deep-lying geometrical interactions, which prevent this particular configuration from being possible.

It is these HIDDEN INTERACTIONS, which prevent small modules from being arranged to form certain desirable larger configurations, that are the essence of this article.

4. PYTHAGOREAN TRIANGLES

These first three examples create a vague intuitive sense that geometry is more complex than we imagine, and that modular elements, cannot always be used to create a given configuration.

Let us now consider a ~~more~~ straightforward example, which is closer, to the problem of arranging office furniture.

Suppose that we wanted to create a certain storeroom in our office layout, which is a particular right angled triangle. To do it, we must build ALL THREE sides of this triangle with available panels, and further, since it is a storeroom, the panels must meet exactly at the corners. (This example does not correspond to any realistic situation. It is presented only to show the mathematical difficulties involved).

We specify the triangle, by giving the lengths of the two short sides. Let us say the overall office layout requires a triangular storeroom whose two short sides are 8 feet and 12 feet. Since we are working with a panel system, we MUST build this triangle out of modular panels.

First, to be most simple, let us suppose that we have just one size of panel, which is four feet long. Obviously, we cannot even build the two short sides of the triangle. The nearest we can come, is either to make both of them 12 feet, or else to make one 8 feet and the other 12.

THE TWO POSSIBLE CONFIGURATIONS.

But even in these two cases, we cannot even BEGIN to get the third side of the triangle right. In the case of the 8-12 triangle, the third side should be 14.42 feet. The nearest we can get with our 4 foot module, is either 12 or 16. In the case of the 12-12 triangle, the third side should be 16.97 feet. The nearest we can get is either 16 or 20.

BOTCHED CONFIGURATIONS

What all this means, is quite simple. With our four foot panel, we cannot even approach the configuration of the triangle we are trying to get.

Let us now try a more realistic approach.

In the Haworth panel system, we have panels of 48", 32", 24" and 16". What this means, is that we actually have a module of 8 inches, since any multiple of 8" can be created by combining the available panels.

With this system, and its 8 inch module, we can get both 10 and 12 feet exactly. We can therefore construct the two short sides of the triangle perfectly.

Now, in order to close the triangle, thre third side, the hypotenuse, must be 15.62 feet.

With our 8" module, we can get 15'4" or 16'0". Each one is about 4 inches off.

But since the system has to close, a miss is as good as a mile. The panels just cannot create the third side of the triangle.

It cannot be done.

-o0o-

Let us ask, just which triangles CAN be created by a modular system.

The answer is given by elementary mathematics.

Any right angled triangle must satisfy the theorem of Pythagoras. In order to be made of modules, this means that each of the three sides, must be an integer. These are the so called pythagorean integral triangles: triangles, whose sides are whole numbers, ~~that~~ satisfy Pythagoras' theorem.

The simplest smallest pythagorean) triangle ~~with integral sides~~ is the famous 3-4-5 triangle.

THE 3-4-5 TRIANGLE

If we have an eight inch module, we can construct a 3-4-5 triangle, in multiples of 8 inches. For instance, we can make a triangle of 2 feet, 2 foot 8", and 3 foot 4 ". Or, coming up to the scale of our storeroom, we can get a triangle of 10 feet, 13'4", and 16'8".

Well, under some cirucumstances, this is not a bad match. But let us assume that we must have the two short sides exactly 10' and 12', otherwise it wont fit into the office layout. This means, we must have a triangle of the shape we saw before. A scaled up 3-4-5 triangle wont do. Then we arent even close.

Can such a triangle be built with ANY modular system, even if the module is very tiny.

Well, yes of course it can.

We must find one of the (larger) pythagorean ~~integer~~ triangles, ~~which has~~ whose short sides are in the ratio 10-12, (and which also has a~~n integral~~ whole number of modules in its hypotenuse)

The smallness of the module we need to accomplish this task, depends on how fussy we are about the match to the 10-12 ratio.

For example, suppose we say that the 10-12 ratio, must be kept within 1%. Then we can find the first pythagorean integer triangle, whose two small sides have a ratio within 1% of 10/12. It is a huge triangle, with sides on the order of 10,000, 12,000 and 15,600.

We may then declare that ~~this~~ this 1% level of approximation to the proper shape of the storeroom can be attained by a module small enough so that there are 10,000 modules in 10 feet -- in short, a module of about .012 inches or one hundredth of an inch.

Let us note that this problem is MATHEMATICAL, not practical.

If we wish to make a storeroom of the specified shape, using modules, THEN IT IS A MATHEMATICAL FACT that the largest module which will accomplish this is about 1/100 inch.

In this case, the modular ratio, the ratio of the key dimensions of the configuration, to the size of the module, is about 10,000 to 1.

If we wish to make triangular configurations within a modular system, then the size of modules we can tolerate, in order to get a reasonable match to the triangles we want, IS VERY VERY SMALL INDEED.

~~Please note, that~~ Of course we are not claiming that triangular configurations ~~play an important role~~ should exist in office layouts. ~~In fact, we believe that they are extremely unlikely to be useful~~.

We are merely demonstrating that the numerical and geometrical interactions between dimensions, even in such a simple layout problem, CAN EASILY create conditions that make modular design impossible, unless the module is very very tiny, compared with the size of the configurations key dimensions.

5. THE HUMAN NOSE.

Let us now leave the abstract subject of geometry, and concentrate on a practical problem of configuration in a simple biological entitity. For fun, and for the sake of example, we take the case of a human nose.

Let us consider the difficulties we would run into, if we were to try to build up a human nose, out of modular entities.

To simplify the matter, let us just consider the surface of the nose. The surface is a curved surface, in three dimensions. It has a highly complex shape. It swells, contracts, has bulges, narrows, nostrils, the bone of the ridge, and so on.

If we were to examine the real cells out of which this complex surface is made, we would find two things. First of all, the cells are extremely small, compared with the size of the nose's global configuration. The nose and its features are on the order of millimeters and centimeters (M^{-3} and M^{-2}). The cells, on the other hand, are on the order of a thousandth of a millemeter across. Thus the cells are on the order of one few thousandth of the size of the main configuration features of the nose.

And secondly, even with this restriction, the cells are NOT modular. They are irregular in shape (like potatoes, pears, etc) and they also vary in dimension. Each one is just the size it needs to be, to fit into its place on the complex curved surface of the nose. No two are exactly the same size.

Now this feature of the way the nose's surface is made of irregular, dimensionally varying cells, is not an accidental feature of the thing. It is MATHEMATICALLY NECESSARY.

It is not hard to see that it is mathematically impossible to construct a surface like the noses surface, from modular entities.

Consider a particular part of the surface shown below.

If we construct a series of adjacent arcs, lying in the surface of the nose, each arc has a different length. Furthermore, the rate at which the length of these arcs changes, is different at different places, according to the degree of curvature which exists in the surface, both in the direction of the arc, and in the direction at right angles to the arc.

This means that it would be quite impossible to construct these arcs, out of little modular beads. Suppose we choose the size of beads, so that one particular arc has a whole number of beads in it. There is then virtually no chance at all, that the next arc (the one which lies one bead diameter away on the surface) WILL ALSO HAVE A LENGTH THAT IS AN INTEGRAL NUMBER OF BEADS.

The reader may point out that at SOME level there must indeed be modules. Perhaps, in this case, at the molecular level.

This, of course, is true. But the difference between the grain size (the key dimensions of the configuration) and the size of the module, is then several orders of magnitude -- perhaps as much as 1:1,000,000.

In this case, the lumpiness of the surface, is so tiny with respect to the global aspects of the nose configuration, that it seems smoothed out, and disappears within the roughness of the surface.

-o0o-

The cells, which can vary infinitely in their size and shape, solve this problem. They allow the curved surface of the nose to be exactly what it has to be, to cover the tissues, and cartilege, and to allow the nose to function.

We note that although this problem seems biological, deeply it is mathematical in nature.

The impossibility of covering a nose with crude modules, is a MATHEMATICAL impossibility. The way that the nose is built out of infinitely varying cells, is based on MATHEMATICAL necessity.

PART 2: OFFICE EXAMPLES

We have looked at a few simple examples, of cases where certain global configurations cannot easily be realised by the use of modules -- or where, in any case, the modules have to be VERY SMALL INDEED, compared with the size of the configurations.

Of course, so far we have not studied any significant examples of office layout, to see if they are similar.

Indeed, the key question is this.

Are the examples we have been looking at "trick" examples, which represent extremely unusual special cases. Or, is it rather the rule that ANY complex configuration in three dimensions, requires a ratio of module to key dimension which is very very small.

We assert the latter.

That is, we assert the following, which we shall call the NON-MODULARITY THEOREM.

WITH THE EXCEPTION OF AN INSIGNIFICANTLY SMALL NUMBER OF SPECIAL CASES, ANY COMPLEX CONFIGURATION IN THREE DIMENSIONS WHICH CONTAINS IMPORTANT DIMENSIONS AND RELATIONS THAT CANNOT BE ARBITRARILY VARIED WILL REQUIRE THE USE OF MODULES WHOSE RATIO IS BETWEEN 1:100 AND 1: 1000 OF THE KEY DIMENSIONS IN THE CONFIGURATION.

This implies that in a well-functioning office layout, whose key configuration dimensions are on the order of 1-4 meters, the largest module which can allow successful realisations, will be between 1 millimeter and 1 centimeter.

One dimensional cases.

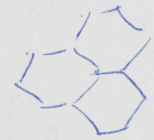
wall. wall + door

→ Table

→ Jesses counter.

Floor boards.

$N + 2N - 3N =$



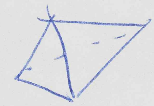
Two dimensional cases.

9" example.

Tom Hewie entrance.

Try Street Rotation on 8" module.

$4 + 4 - 6 = 2$



Three dimensional cases.

→ Star Card.

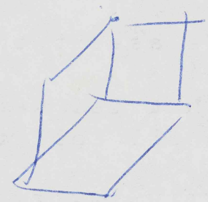
Rodneys wall.

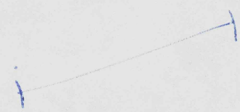
Two walls meet

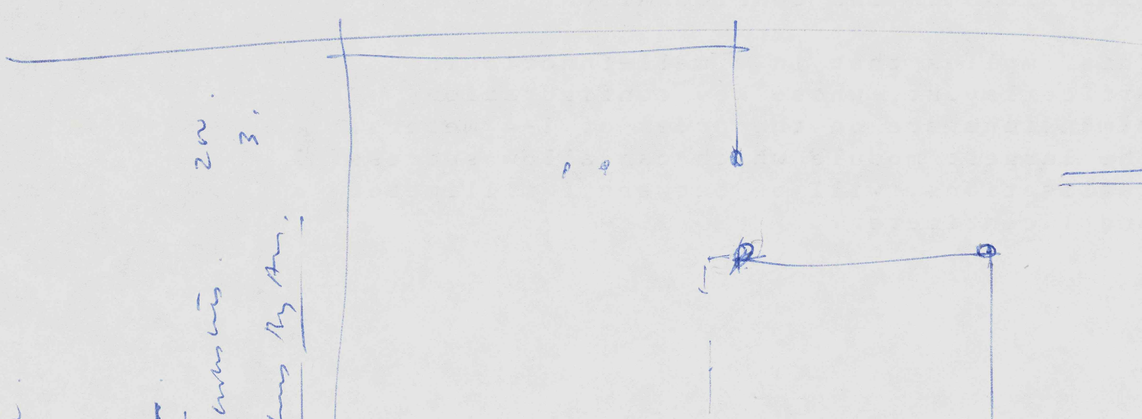
Slope of the desk

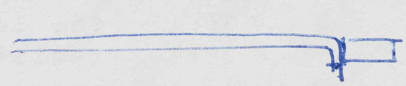
$6 + 8 - 12 = 2$

$F + V - E$









30m/ locali

6 m

30m, m

50 m. ...

(12)