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DEPARTMENT OF CIVIL ENGINEERING

CIVIL ENGINEERING SYSTEMS LABORATORY

HIDECS 3: FOUR COMPUTER PROGRAMS FOR THE HIERARCHICAL DECOMPOSITION OF SYSTEMS WHICH HAVE AN ASSOCIATED LINEAR GRAPH

by

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INTRODUCTION

Confronted by a system of elements, some linked to one another, others not linked, the most natural question to ask about the system is: "What are its subsystems".

The question is by no means a simple one; first of all, it all depends on what you mean by "subsystem". Each of the four programs described in this report is based on a different conception or definition of a "subsystem".

These programs, each on the basis of a different definition of 'subsystem', determine the decomposition of any system into its subsystems. In every case the system is taken to be defined by a set of elements, M, and a set of two-element links, L. The system is therefore completely described by a graph G(M,L); and is represented in the computer by a binary matrix.

The input, machine representation, and output, for all four programs are the same as in the program HIDECS 2, described in a previous report in this series. All the supporting subprograms like GENER, COUNT, CNVRT, etc, are also the same as in HIDECS 2. Each of the programs to be described in this report, namely BLDUP, STABL, SIMPX, EQCLA, is a core subprogram, under the control of MAIN thus similar in content and function to LGRMN, the core subprogram of HIDECS 2.

The actual system decompositions defined by these programs is however quite different; each one is intended to take care of certain weaknesses in HIDECS 2. So that this report may be selfcontained, we begin with a brief summary of HIDECS 2.

In HIDECS 2 each element of M is assumed to be a binary stochastic variable. The decomposition of the system into subsystems is specified by a tree of subsets of M. At the top level of the tree is the set M. At the second level there are two disjoint subsets of M, whose union is M. At each new level in the tree, every subset is broken into two further disjoint subsets. The problem is to find that tree which is most appropriate in view of the linkages defined by L, and which thus succeeds in isolating what we should most want to call the subsystems of the system.

It has been shown (Alexander, 1963a) that one way of doing this is to define the information transfer between sets of variables. At each level of the tree, a set of variables is broken into those two of its subsets with minimum information transfer between them. Let m be the number of variables in M, ℓ the number of links in L, s_1 and s_2 the numbers of variables in the two subsets ($s_1 + s_2 = m$), and r the number of links between the two subsets. The information transfer, corrected for bias toward special partitions, is:

$$\text{INFO} = \frac{\left\{v - \left[\frac{2l}{m(m-1)}\right] s_1 s_2\right\}^2 \delta}{s_1 s_2 \left[\frac{m(m-1)}{2} - s_1 s_2\right]} ,$$

where δ is +1 or −1 according as the top bracket is positive or negative.

The program HIDECS 2 uses this function, INFO, as the criterion for a steepest-ascent hill-climbing procedure. The program begins by generating a random pair of complementary subsets of M. It then tests all pairs of subsets which can be derived from this pair by shifting a single element from one subset to the other. The best pair of subsets replaces the starting pair. The program repeats this process of testing and replacement until it finds a pair of subsets which cannot be improved by shifting a single element. Experiments show that the number of independent hill-climbs required to reach absolute optimum is small.

HIDECS 2 has three important weaknesses:

1. The fact that the decomposition is made in a series of binary steps leads to certain 'mistakes', since the holistic relatedness of system and subsystems is not properly taken into account.
2. The fact that the decomposition criterion INFO is based on very stringent assumptions about the nature of the system G(M,L). Namely, that the elements of M are binary

variables, that the two variable correlations are very small, and that the many variable correlations vanish altogether. These assumptions make it hard to find systems in the real world which the formalism of HIDECS 2 can adequately represent.

3. The fact that the subsets of elements which make the most natural subsystems of a system are not always disjoint, but often overlap.

In the four programs to be described, these weaknesses are overcome as follows:

In BLDUP, the decomposition criterion, though still essentially the same as that used in HIDECS 2, has been extended so that not only 2-way, but 3-way, 4-way, etc. partitions can all be compared with one another. This means that the decomposition into subsystems need not be defined stepwise, but can be defined all at once, and the holistic nature of the system thereby better preserved.

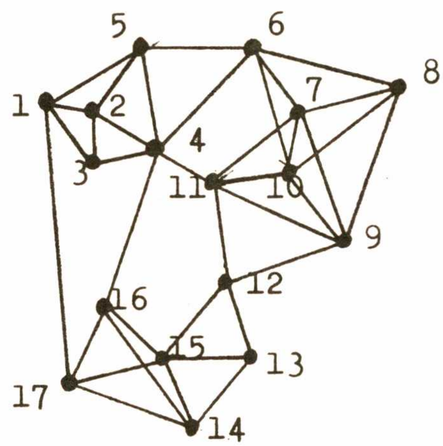
In STABL, SIMPX, and EQCLA the elements of the system are no longer assumed to be binary variables, or indeed variables at all. The elements of M may be elements of any kind, and the links between elements, though still only permissible between two elements at a time, may be of any sort whatever. In all three cases, as in BLDUP, the subsystems are defined simultaneously, not sequentially.

Finally, in SIMPX and EQCLA, the subsystems are defined in such a way that they overlap instead of being disjoint.

In fact, in these two cases the decomposition, instead of being a tree, is a lattice.

In the following four sections we shall examine the four subprograms BLDUP, STABL, SIMPX, EQCLA, in detail.

So that the content of each of the four programs may become intuitively clear, each section will close with decomposition of the following graph:



The decomposition which HIDECS 2 gives of this graph, is as follows:

1	2	3	4	5	0	0	0	0	0	0	0	13	14	15	16	17
0	0	0	0	0	6	7	8	9	10	11	12	0	0	0	0	0

NEW LEVEL OF HIERARCHY

1	2	3	4	5	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	13	14	15	16	17
0	0	0	0	0	0	0	0	0	0	11	12	0	0	0	0	0
0	0	0	0	0	6	7	8	9	10	0	0	0	0	0	0	0

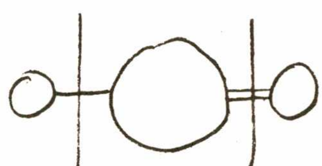
NEW LEVEL OF HIERARCHY

0	2	0	4	5	0	0	0	0	0	0	0	0	0	0	0	0
1	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	14	15	16	17
0	0	0	0	0	0	0	0	0	0	0	0	13	0	0	0	0
0	0	0	0	0	0	0	0	9	0	0	0	0	0	0	0	0
0	0	0	0	0	6	7	8	0	10	0	0	0	0	0	0	0

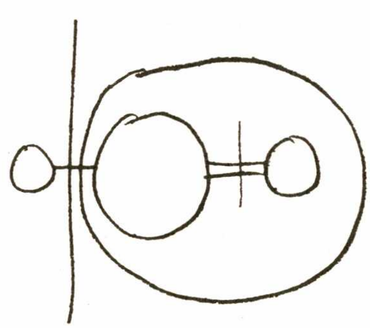
BLDUP

Before describing an n-way partition of M, let us explore the theory of achieving multiple decomposition by successive binary partitions. The purpose of partitioning the set M is to obtain as complete a description of the system's structure as we can. If we make a practice of partitioning into n subsystems at each step, we should, in general, introduce spurious regularities into our description. This is because the best n-way partition will not, in general, be entirely regular, some of the partitioned sets will be more strongly related to one another than others. Thus, for example, take the following simple case:

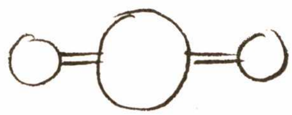
It might be argued that this is best described by a 3-way partition. Thus:



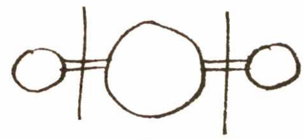
But the left-hand subsystem is less strongly connected to the center than the right hand one. If we described this by means of a 3-way partition we should be ignoring this assymetry or inequality. We shall have a more accurate picture of the structure if we describe it by means of two 2-way partitions. Thus:



This kind of description fails only in that case where the linkages are exactly equal, as in:



In this case we must write

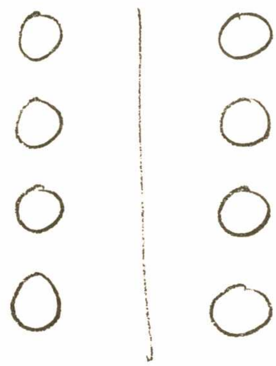


because we have no grounds for introducing any assymetry. But in this case, as is easy to see, there will be two 2-way partitions of equal strength, which together divide the set into 3 subsets. Thus:

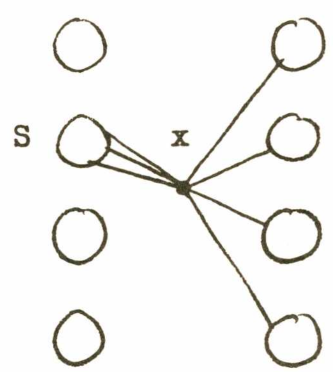
Indeed, it is not hard to convince oneself that in every case where an n-way partition is the best way to describe a decomposition, there are just the right number of equal best 2-way partitions to give that n-way partition in concert.

However, the defect of any algorithm which partitions M into two subsets at a time, is that it does not pay sufficient attention to the gestalt, or overall pattern of the subsystems, and therefore introduces a bias which by any reasonable intuition is a 'mistake'.

Consider an example. Suppose we have a system which, at the first level of decomposition, is to be divided into two subsystems, as shown below, leaving a situation which further partitioning will then split further into the four subsystems on the left, and four subsystems on the right.



Suppose now that there is one element x which we are in doubt about how to place in this first partition. Let us say that it is connected by 1 link to each of the four subsystems on the right; and that it is connected by 3 links to just one of the subsystems on the left, and to none of the others on the left.



Other things being equal, a two-way partition algorithm will, at this level of decomposition, assign this point to the right hand subsystems rather than to the left hand - because it is linked to the right by a total of 4 links and by a total of only 3 links to the left. Yet the outcome of this decision, when the decomposition proceeds to lower levels of decomposition, is that this element will be associated to one of the subsystems on the right, to which it has but 1 link.

The most appropriate subsystem for this element to belong to is of course the subsystem, S, to which it is connected by 3 links. But the two-way partition algorithm is unable to assign the element x intelligently because, as it were, it has no way of seeing into the future lower levels of decomposition not yet carried out. This has actually happened, in real analysis under HIDECS 2, and has led to irritating anomalies. To avoid it, we must make use of an algorithm which surveys the entire decomposition all at once.

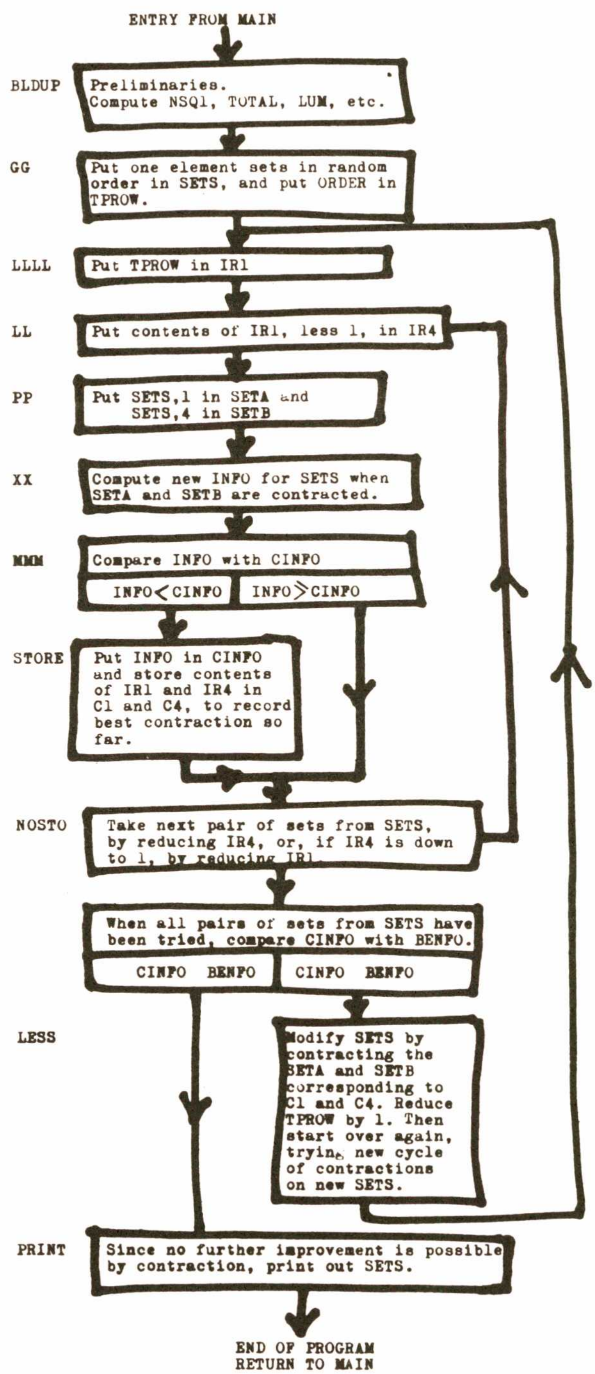
For any decomposition of M into disjoint sets S_1, S_2,..., define

$$\mathrm{INFO} = \frac{\left\{ r - \frac{2l}{m(m-1)} \sum_{i \neq j} s_i s_j \right\}^2 \delta}{\sum_{i \neq j} s_i s_j \left(\frac{m(m-1)}{2} - \sum_{i \neq j} s_i s_j \right)}$$

where s_i is the number of elements in S_i, and δ is 1 or -1 according as the bracket is positive or negative. The task of BLDUP is to find that decomposition of M into disjoint sets, which has the minimum value of INFO.

The program is a hill-climbing program. The basic operational unit of the hill climb is the contraction of a decomposition, in which two sets of the decomposition S_i and S_j, are joined to form a single set $S_i \cup S_j$, thus giving another decomposition. The hill climb begins by placing the M one-element sets, in numerical order, in SETS. This defines the unit decomposition of M into its individual vertex sets. The program examines each possible contraction of the unit decomposition, computing INFO for each contraction, and then forms that contraction whose value of INFO is least. It then continues the same process, computing INFO for all the contractions of this new decomposition, and again forming whichever contraction is the best; the process of contraction is repeated until the program reaches a decomposition which has a lower value of INFO than any of its contractions; at this point no improvement is possible by contraction: The hill climb terminates, and the program prints out the contents of SETS.

Although it is possible that there might be decompositions, not reachable by this process of contraction, with even lower values of INFO, experiments have shown that this is not usually the case; the reason seems to be that the function INFO varies only very slowly with changes in decomposition.


ENTRY FROM MAIN
BLDUP
Preliminaries.
Compute NSQ1, TOTAL, LUM, etc.
GG
Put one element sets in random order in SETS, and put ORDER in TPROW.
LLLL
Put TPROW in IR1
LL
Put contents of IR1, less 1, in IR4
PP
Put SETS,1 in SETA and SETS,4 in SETB
XX
Compute new INFO for SETS when SETA and SETB are contracted.
MMM
Compare INFO with CINFO
INFO<CINFO
INFO>CINFO
STORE
Put INFO in CINFO and store contents of IR1 and IR4 in C1 and C4, to record best contraction so far.
NOSTO
Take next pair of sets from SETS, by reducing IR4, or, if IR4 is down to 1, by reducing IR1.
When all pairs of sets from SETS have been tried, compare CINFO with BENFO.
CINFO BENFO
CINFO BENFO
LESS
Modify SETS by contracting the SETA and SETB corresponding to C1 and C4. Reduce TPROW by 1. Then start over again, trying new cycle of contractions on new SETS.
PRINT
Since no further improvement is possible by contraction, print out SETS.
END OF PROGRAM
RETURN TO MAIN

```
*     FAP

                    CCC05          ENTRY   BLDUP


  TRANSFER VECTOR
CCC00  234664456360      COUNT
00001  47634365436C      PTLVL
00002  47636225636C      PTSFT

  LINKAGE DIRECTOR
00003  000000000C0C
00004  22432464476C

00005 -0634 00 4 C0452    BLDUP SXD IR4,4
                         *     PRELIMINARIES
00006  0500 C0 0 77455          CLA LATIS
00007  0622 C0 0 C0455          STD NDXX
CC010 -0534 00 I 77457    AA    LXD DAT,1
00011 -0500 C0 1 76336          CAL DROWS,1
00012  0602 00 I 72252          SLW MROWS,1
00013  2 00001 1 CCC11          TIX *-2,1,1
                         *    COMPUTE NSQ1
00014  0500 00 0 77461    BB    CLA ORDER
C0015  0402 00 0 77444          SUB CNED
C0016 -0765 00 0 CCC44          LGR 36
C0017  0200 00 0 77461          MPY ORDER
00020  0767 C0 0 0CC20          ALS 16
00021  0622 00 0 77450          STD NSQ1
                         *   COMPUTE  TOTAL
00022 -0534 00 1 77457    CC    LXD DAT,1
00023  0600 00 0 C2135          STZ TOTAL
00024  0500 00 0 C2135          CLA TOTAL
00025  0560 C0 1 72252          LDQ MROWS,1
00026  0522 60 0 CCC00          XEC* $COUNT
00027  0622 00 0 C2135          STD TOTAL
00030  2 00001 I CCC24          TIX *-4,1,1
00031  0500 C0 0 02135          CLA TOTAL
C0032  0771 C0 0 C0001          ARS 1
00033  0622 00 0 C2135          STD TOTAL
                         *   COMPUTE LUM
00034  0560 C0 0 C2141    DD    LDQ =0
00035  0220 00 0 77450          DVH NSQ1
00036  0760 00 0 CCC00          CLM
00037 -0765 00 0 0CC21          LGR 17
C0040 -0600 C0 0 C0453          STQ LUM
                         *   PREPARE TO START
                         * NEXT SECTION GENERATES INDIRECT
                         *   ADDRESSING BEHIND SETS
00041  0500 C0 0 77461          CLA ORDER
C0042  0622 C0 0 C0241          STD S902
00043  0500 00 0 CCC62    FF    CLA MXM1
C0044  0602 C0 0 01473          SLW SETS-1
00045  0500 C0 0 77461          CLA ORDER
00046  0622 00 0 CCC60          STD LOC
00047  0500 00 0 77460          CLA NWORD
00050  0400 00 0 77444          ACD CNED
C0051  0771 00 0 CCC22          ARS 18
```

```
00052  0601 00 0 CCC63          STO DIFSP
C0053  0774 00 I CC001          AXT 1,1
00054  0500 00 1 01474          CLA SETS,1
00055  0402 00 0 CCC63          SUB DIFSP
00056  1 00001 I CCC57          TXI *+1,1,1
00057  0602 CO I 01474          SLW SETS,1
00060 -3 00000 I CCC55    LOC   TXL *-3,1,**
C0061  C020 00 0 CCC64          TRA *+3
C0062  CC0000247616       MXM1  VFD 18/0,03/2,15/MACRO
C0063  0 CCCCO 0 CCC00    DIFSP PZE
00064  0761 CO 0 CCC00          NOP
00065  0500 00 0 77461          CLA ORDER
C0066  0622 00 0 CCC77          STD TXL1
                          * NOW BEGINS SECTION TO GENERATE NOS
00067  C074 CO 4 CC001    GG    CALL PTLVL
C0070  1 C0000 0 CCC72
C0071  0 00505 0 CCC03
C0072  0774 00 I CC001          AXT 1,1
00073  0500 00 0 77444          CLA CNED
00074  0622 00 1 C2134          STD TABLE,1
00075  0400 00 0 77444          ADD CNED
00076  1 C0001 1 CCC77          TXI *+1,1,1
00077 -3 00000 I CC074    TXL1  TXL *-3,1,**
00100  0020 00 0 CC101          TRA HH
                          * GENERATE RANDOM ORDER
00101 -0534 CO 4 77461    HH    LXD ORDER,4
00102 -0534 00 1 77461    START LXD ORDER,1
00103 -0500 CO 0 77404    GEN   CAL RANDM
00104  0361 00 0 77372          ACL DIFF
00105  0602 CO 0 77404          SLW RANDM
00106  0500 00 0 77461          CLA ORDER
00107 -0320 00 0 77404          ANA RANDM
C0110  0622 00 0 CC111          STD TIX
C0111  2 000CO 1 C0113    TIX   TIX *+2,1,**
00112  0020 00 0 C0102          TRA START
00113  0500 CO 1 C2134          CLA TABLE,1
00114 -0100 00 0 CC120          TNZ MOVE
00115  2 0C001 I CC113          TIX *-2,1,1
00116 -0534 00 I 77461          LXD ORDER,1
00117  0020 00 0 CC113          TRA *-4
00120  0622 CO 4 C1C62    MOVE  STD NOS,4
C0121  0600 00 1 C2134          STZ TABLE,1
00122  2 00001 4 C0111          TIX TIX,4,1
00123  0020 00 0 CC124          TRA JJ
                          * MOVE SETS INTO RANDOM PLACES
00124 -0534 CO I 77457    JJ    LXD CAT,1
00125  0600 00 I 47616          STZ MACRO,1
00126  2 00001 1 CC125          TIX *-1,1,1
00127 -0534 00 I 77461          LXD ORDER,1
00130  0500 CO I 01062    LOAD  CLA NOS,1
00131 -0734 00 4 CC000          PDX C,4
00132  0774 00 2 CCC01          AXT 1,2
C0133  2 00044 4 0C140    DOWN  TIX STEP,4,36
C0134 -0500 CO 4 77170          CAL UNIT,4
00135  0602 60 I 01474          SLW* SETS,1
00136  2 C0001 1 0C130          TIX LOAD,1,1
```

```
  00137  0020 00 0 CC142          TRA KK
  00140  1 00001 2 CC133   STEP   TXI DOWN,2,1
A 00141  0C00 C0 0 CCCC0          HTR
  00142  0500 00 0 77450   KK     CLA NSQ1
  00143  0622 C0 0 C1C67          STD SSSUM
  00144  0500 C0 0 C2135          CLA TOTAL
  00145  0622 C0 0 C1C66          STD RR
  00146  0500 00 0 C2142          CLA =C377777777777
  00147  0601 00 0 CC443          STO CINFO
  C0150  0601 00 0 CC424          STO BENFO
  00151 -0534 C0 1 77461          LXD ORDER,1
  00152 -0634 00 1 C1527   LLLL   SXD TPROW,1
  00153 -0754 00 1 CCC00   LL     PXD 0,1
  00154  0402 00 0 77444          SUB CNED
  C0155 -0734 C0 4 CCCC0          PDX C,4
  C0156 -0534 00 2 77460   PP     LXD NWORD,2
  00157 -0500 60 1 C1474          CAL* SETS,1
  00160  C602 C0 2 01507          SLW SETA,2
  00161  2 00001 2 CC157          TIX *-2,2,1
  00162 -0534 00 2 77460          LXD NWORD,2
  00163 -0500 60 4 C1474          CAL* SETS,4
  00164  C602 C0 2 C1522          SLW SETB,2
  0813   2 8801 2 C0163        TIX *-2Z ,1
  C0166  0020 00 0 C0167          TRA XX
                           * NEXT SECTION COMPUTES INFO
  00167 -0634 00 1 C2137   XX     SXD XY1,1
  C0170 -0634 00 4 C214C          SXD XY4,4
  00171  0600 C0 C C1523          STZ SA
  00172 -0534 00 2 7746C          LXD NWORD,2
  00173  0500 00 0 C1523          CLA SA
  00174  0560 C0 2 01507          LCQ SETA,2
  00175  0522 60 0 CCCC0          XEC* $COUNT
  00176  0622 00 0 C1523          STD SA
  00177  2 C0001 2 CC173          TIX *-4,2,1
  C0200  0600 00 C C1524          STZ SB
  C0201 -0534 C0 2 77460          LXD NWORD,2
  00202  0500 00 0 01524          CLA SB
  00203  0560 C0 2 C1522          LCQ SETB,2
  00204  0522 60 C CCC0C          XEC* $COUNT
  00205  0622 00 0 C1524          STD SB
  00206  2 00001 2 00202          TIX *-4,2,1
  00207  0500 00 0 C1523          CLA SA
  C0210 -0765 00 0 CCC44          LGR 36
  C0211  0200 00 0 01524          MPY SB
  00212  0767 00 0 CC021          ALS 17
  00213  0622 00 0 C1525          STD SASB
  00214  0600 C0 0 C0423          STZ ARB
  00215  0774 C0 4 CCC01          AXT 1,4
  00216  0774 00 I CCC01          AXT 1,1
  00217  0560 00 4 C1507   LDTST  LCQ SETA,4
  00220 -0634 C0 4 CI526          SXD SXRD,4
  00221  0774 C0 4 CCC01          AXT 1,4
  00222  0162 00 0 C0236   TQPP   TQP SKIP
  00223 -0600 00 0 C2136          STQ WAIT
  00224 -0534 00 2 77460   AROW   LXD NWORD,2
  00225 -0500 C0 2 C1522          CAL SETB,2
```

```
00226 -0320 60 1 76742          ANA* MATAX,1
00227  0100 00 0 00234          TZE **+5
00230 -0765 00 0 00044          LGR 36
00231  0500 00 0 00423          CLA ARB
00232  0522 60 0 00000          XEC* $COUNT
00233  0622 00 0 00423          STD ARB
00234  2 00001 2 00225          TIX AROW+1,2,1
00235  0560 00 0 02136          LDQ WAIT
00236 -0773 00 0 00001    SKIP  RQL 1
00237  1 00001 1 00240          TXI **+1,1,1
00240  1 00001 4 00241          TXI **+1,4,1
00241  3 00000 1 00245    S902  TXH RRRR,1,**
00242 -3 00044 4 00222          TXL TQPP,4,36
00243 -0534 00 4 01526          LXD SXRD,4
00244  1 00001 4 00217          TXI LDTST,4,1
00245  0020 00 0 00246    RRRR  TRA YY
                          * YY ACTUALLY DOES THE DIVISION
00246  0500 00 0 01066    YY    CLA RR
00247  0402 00 0 00423          SUB ARB
00250  0622 00 0 01063          STD NRR
00251  0500 00 0 01067          CLA SSSUM
00252  0402 00 0 01525          SUB SASB
00253  0622 00 0 01064          STD NSSUM
00254  0500 00 0 77450          CLA NSQ1
00255  0402 00 0 01064          SUB NSSUM
00256  0765 00 0 00043          LRS 35
00257  0200 00 0 01064          MPY NSSUM
00260  0771 00 0 00001          ARS 1
00261  0601 00 0 00425          STO BOTT
00262 -0100 00 0 00265          TNZ **+3
00263  0600 00 0 00451          STZ INFO
00264  0020 00 0 00303          TRA TAGET
00265  0500 00 0 01064          CLA NSSUM
00266  0765 00 0 00043          LRS 35
00267  0200 00 0 00453          MPY LUM
00270  0763 00 0 00021          LLS 17
00271  0402 00 0 01063    SAD   SUB NRR
00272  0760 00 0 00002          CHS
00273  0601 00 0 00454          STO MULT
00274  0120 00 0 00276          TPL **+2
00275  0760 00 0 00002          CHS
00276  0765 00 0 00043          LRS 35
00277  0200 00 0 00454          MPY MULT
00300  0765 00 0 00001          LRS 1
00301  0220 00 0 00425          DVH BOTT
00302 -0600 00 0 00451          STQ INFO
00303 -0534 00 1 02137    TAGET LXD XY1,1
00304 -0534 00 4 02140          LXD XY4,4
00305  0020 00 0 00306          TRA MMM
                          * END OF SECTION WHICH COMPUTES INFO
                          * NEXT SECTION COMPARES INFO WITH CINFO
00306  0500 00 0 00451    MMM   CLA INFO
00307  0340 00 0 00443          CAS CINFO
00310  0020 00 0 00330          TRA NOSTO
00311  0020 00 0 00330          TRA NOSTO
00312  0020 00 0 00313          TRA STORE
```

```
00313 -0634 00 4 00426   STORE SXD C4,4
00314 -0634 00 1 00427         SXD C1,1
00315 -0534 00 2 77460         LXD NWORD,2
00316 -0500 00 2 01507         CAL SETA,2
00317 -0501 00 2 01522         ORA SETB,2
00320  0602 00 2 00442         SLW CET,2
00321  2 00001 2 00316         TIX *-3,2,1
00322  0500 00 0 01064         CLA NSSUM
00323  0601 00 0 00444         STO CSSUM
00324  0500 00 0 01063         CLA NRR
00325  0601 00 0 00445         STO CRR
00326  0500 00 0 00451         CLA INFO
00327  0601 00 0 00443         STO CINFO
00330  2 00001 4 00156   NOSTO TIX PP,4,1
00331 -3 00002 1 00333         TXL *+2,1,2
00332  2 00001 1 00153         TIX LL,1,1
00333  0500 00 0 00443         CLA CINFO
00334  0340 00 0 00424         CAS BENFO
00335  0020 00 0 00366         TRA PRINT
00336  0020 00 0 00366         TRA PRINT
00337  0020 00 0 00340         TRA LESS
00340 -0534 00 4 00426   LESS  LXD C4,4
00341 -0534 00 1 00427         LXD C1,1
00342 -0534 00 2 77460         LXD NWORD,2
00343 -0500 00 2 00442         CAL CET,2
00344  0602 60 4 01474         SLW* SETS,4
00345  2 00001 2 00343         TIX *-2,2,1
00346  0500 00 0 00444         CLA CSSUM
00347  0601 00 0 01067         STO SSSUM
00350  0500 00 0 00445         CLA CRR
00351  0601 00 0 01066         STO RR
00352  0500 00 0 00443         CLA CINFO
00353  0601 00 0 00424         STO BENFO
00354 -0534 00 4 01527         LXD TPROW,4
00355 -0534 00 2 77460         LXD NWORD,2
00356 -0500 60 4 01474         CAL* SETS,4
00357  0602 60 1 01474         SLW* SETS,1
00360  0600 60 4 01474         STZ* SETS,4
00361  2 00001 2 00356         TIX *-3,2,1
00362 -0534 00 1 01527         LXD TPROW,1
00363 -3 00002 1 00365         TXL *+2,1,2
00364  2 00001 1 00152         TIX LLLL,1,1
00365  0020 00 0 00420         TRA OUT
00366  0500 00 0 77460   PRINT CLA NWORD
00367  0622 00 0 00446         STD HOLD
00370 -0534 00 2 77457         LXD DAT,2
00371 -0500 00 2 47616         CAL MACRO,2
00372  0602 00 2 66166         SLW INMAT,2
00373  2 00001 2 00371         TIX *-2,2,1
00374 -0534 00 4 00446   FRESH LXD HOLD,4
00375 -0534 00 2 77460         LXD NWORD,2
00376 -0500 00 4 66166         CAL INMAT,4
00377  0602 00 2 77416         SLW SET,2
00400  2 00001 4 00401         TIX *+1,4,1
00401  2 00001 2 00376         TIX *-3,2,1
00402  0500 00 0 00446         CLA HOLD
```

```
00403  0400 00 0 77460         ADD NWORD
00404  0400 00 0 77444         ADD CNED
00405  0622 00 0 C0446         STD HOLD
00406 -0534 00 2 77460         LXC NWORD,2
00407  0760 00 0 C0000         CLM
00410 -0501 00 2 77416         ORA SET,2
00411  2 00001 2 C0410         TIX *-1,2,1
00412  0100 C0 0 C0417         TZE ENDPT
00413  0074 C0 4 CC002         CALL PTSET
00414  1 00000 0 C0416
00415  0 20607 C CCC03
00416  0020 00 0 C0374         TRA FRESH
00417  C020 00 0 00420   ENDPT TRA OUT
00420 -0534 00 4 C0452   OUT   LXD IR4,4
00421  0020 C0 4 CCC01         TRA 1,4
00422  0 CC0C0 0 CCC00         PZE
00423  0 CC000 0 C0000   ARB
00424  0 00000 0 CCC00   BENFO
00425  0 00C00 C CCC00   BOTT
00426  0 CC000 0 C0C00   C4
00427  0 0CC00 0 C0C00   C1
00442                          BES 10
00442  0 00000 C CCC00   CET
00443  0 0C0C0 C CCC00   CINFO
00444  0 0CC00 0 CCCC0   CSSUM
00445  0 00000 0 C0C00   CRR
00446  0 00000 0 CCC00   HOLD
00447  0 C0CC0 0 CCC00   INOPN
00450  0 C0000 0 CCC00   INBEN
00451  0 00000 0 C0000   INFO
00452  0 00000 C CCC00   IR4
00453  0 000C0 0 CCC00   LUM
00454  0 CCCC0 0 CCC00   MULT
00455  0 00000 0 C0C00   NDXX
01062                          BES 260
01062  0 000C0 0 CCCC0   NOS
01063  0 00000 0 CCC00   NRR
01064  0 00000 0 CCC00   NSSUM
01065  0 C0000 0 CCC00   OPNFO
01066  0 COCC0 0 CCC00   RR
01067  0 CCC00 0 CC000   SSSUM
01474                          BES 260
01474  0 00000 C CCC00   SETS
01507                          BES 10
01507  0 00000 0 CC000   SETA
01522                          BES 10
01522  0 00C00 0 CC000   SETB
01523  0 000C0 0 CCC00   SA
01524  0 00000 0 CC000   SB
01525  0 00000 0 CCOC0   SASB
01526  0 00000 C C0C00   SXRD
01527  0 000C0 0 CCC00   TPROW
02134                          BES 260
02134  0 000C0 0 C0000   TABLE
02135  0 00CC0 0 C0C00   TOTAL
02136  0 000C0 0 CCC00   WAIT
```

```
02137   0 00C00 0 CCC00    XY1
02140   0 000C0 0 CC000    XY4
                         *    COMMCN BLOCK FROM HIDECS 2
                77462          COMMCN -1
                77462     INDIC COMMCN 1
                77461     ORDER COMMCN 1
                77460     NWORD COMMCN 1
                77457     DAT   COMMCN 1
                77456     LGTH  COMMCN 1
                77455     LATIS COMMCN 1
                77454     NBITH COMMCN 1
                77453     NBITL COMMCN 1
                77452     NBIT1 COMMCN 1
                77451     NBIT  COMMCN 1
                77450     NSQ1  COMMCN 1
                77447     OPRMN COMMCN 1
                77446     ATOMO COMMCN 1
                77445     ATOM  COMMCN 1
                77444     ONED  COMMCN 1
                77443     D36   COMMCN 1
                77442     ATCOX COMMCN 10
                77430     ATCX  COMMON 10
                77416     SET   COMMCN 10
                77404     RANCM COMMCN 1C
                77372     DIFF  COMMCN 10
                77360     CONVT COMMON 40
                77310     DATA  COMMCN 40
                7724C     MATA  COMMCN 40
                7717C     UNIT  COMMCN 40
                77120     COMUN COMMCN 40
                77C50     EQLS  COMMCN 20
                77C24     SECTS COMMCN 50
                76742     MATAX COMMCN 260
                76336     DROWS COMMCN 2100
                72252     MROWS COMMCN 2100
                66166     INMAT COMMCN 5400
                53536     ATMS  COMMCN 2C00
                47616     MACRO COMMCN 7000
                                END

 LITERALS
02141   00000CCC0C000
02142   377777777777
```

The decomposition of the sample graph follows:

```
 2  3
14 15 16
 4  5  6
12 13
 1 17
 7  8  9 10 11
```

STABL

This program is very similar to BLDUP. For any decomposition of M into disjoint sets $S_1, S_2, \ldots$, define the function EXP as:

$$\text{EXP} = \frac{\left\{ \sum_i \left[l_i \cdot \frac{m(m-1)}{2l} - \frac{s_i(s_i-1)+1}{2} \right] \right\}^2 \delta}{\sum_i \left(\frac{s_i(s_i-1)+1}{2} \right) 2^{-2s_i}}$$

, where δ is $+1$ or -1 according as the top bracket is positive or negative.

The task of STABL is to find that decomposition of M into disjoint sets, for which EXP is maximum. It has been shown elsewhere that the decomposition so obtained is maximally stable under the addition of new and unknown elements to the system M.*

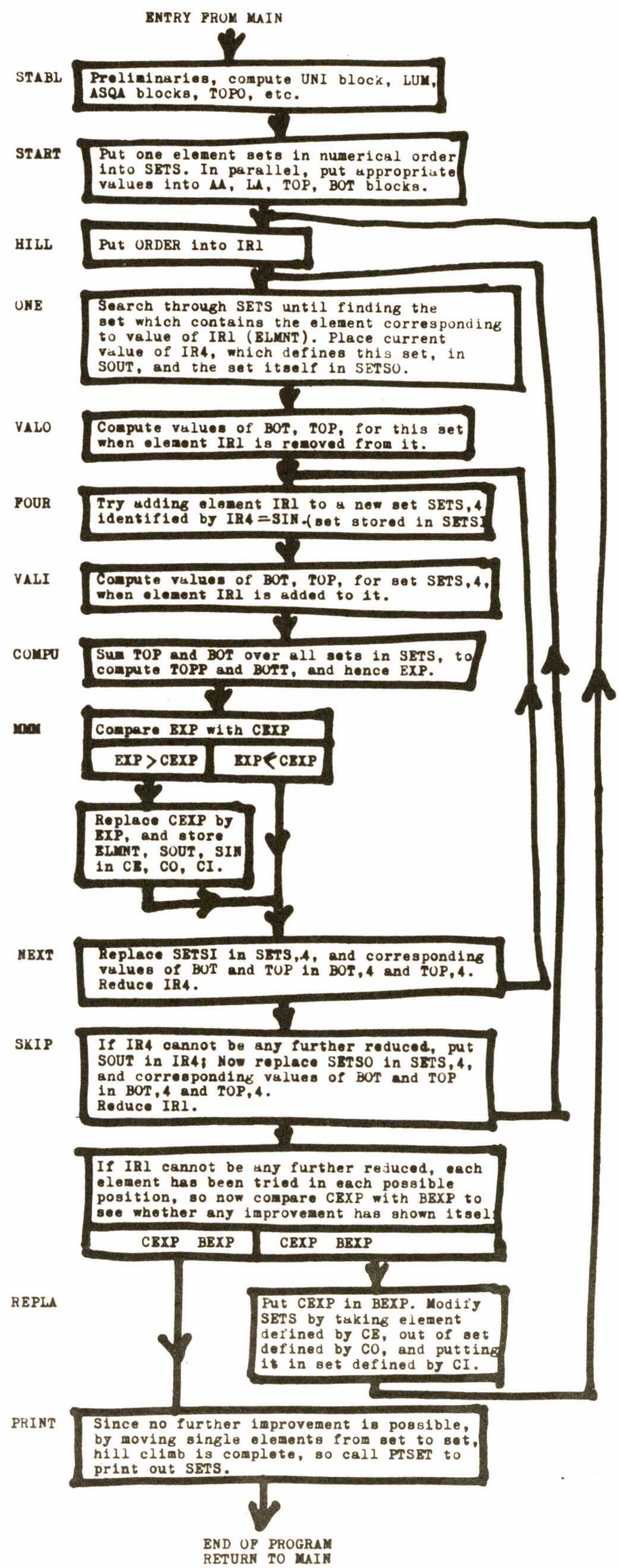
This function EXP varies sharply even over slight variation in the decomposition. The crude hill-climb by successive contractions, used in BLDUP, is therefore unsuitable for STABL. In fact, in experience, even for small and simple systems, a hill-climb based on contraction failed to find the decomposition with the best value of EXP. Instead STABL is based on the following procedure.

Start with the unit decomposition in SETS, as before. The basic operation consists of moving one element, out of the set it happens to be in, and adding it to each of the other sets in turn, computing EXP for each new decomposition so obtained. This is done for each element. The best of

---

*Actually EXP differs slightly from the criterion function given in (1963 b); the changes make it more continuous in the search space; the original function had such severe discontinuities that the hill climb would not work at all. l_i is the number of links in the subsystem S_i.

all the decompositions so obtained is thus the best decomposition to be obtained by moving a single element. The program makes this change; and then begins the cycle again. The program terminates when it finds a decomposition whose value of EXP is higher than that for any decomposition obtainable from it by moving a single element.


ENTRY FROM MAIN
STABL
Preliminaries, compute UNI block, LUM, ASQA blocks, TOPO, etc.
START
Put one element sets in numerical order into SETS. In parallel, put appropriate values into AA, LA, TOP, BOT blocks.
HILL
Put ORDER into IR1
ONE
Search through SETS until finding the set which contains the element corresponding to value of IR1 (ELMNT). Place current value of IR4, which defines this set, in SOUT, and the set itself in SETSO.
VALO
Compute values of BOT, TOP, for this set when element IR1 is removed from it.
FOUR
Try adding element IR1 to a new set SETS,4 identified by IR4 = SIN (set stored in SETSI
VALI
Compute values of BOT, TOP, for set SETS,4, when element IR1 is added to it.
COMPU
Sum TOP and BOT over all sets in SETS, to compute TOPP and BOTT, and hence EXP.
MMM
Compare EXP with CEXP
EXP > CEXP
EXP < CEXP
Replace CEXP by EXP, and store ELMNT, SOUT, SIN in CE, CO, CI.
NEXT
Replace SETSI in SETS,4, and corresponding values of BOT and TOP in BOT,4 and TOP,4. Reduce IR4.
SKIP
If IR4 cannot be any further reduced, put SOUT in IR4; Now replace SETSO in SETS,4, and corresponding values of BOT and TOP in BOT,4 and TOP,4. Reduce IR1.
If IR1 cannot be any further reduced, each element has been tried in each possible position, so now compare CEXP with BEXP to see whether any improvement has shown itsel
CEXP BEXP
CEXP BEXP
REPLA
Put CEXP in BEXP. Modify SETS by taking element defined by CE, out of set defined by CO, and putting it in set defined by CI.
PRINT
Since no further improvement is possible, by moving single elements from set to set, hill climb is complete, so call PTSET to print out SETS.
END OF PROGRAM
RETURN TO MAIN

```
*       FAP

                     00005              ENTRY   STABL

  TRANSFER VECTOR
00000   476343654360        PTLVL
00001   234664456360        COUNT
00002   476362256360        PTSET

  LINKAGE DIRECTOR
00003   000000000000
00004   626321224360


00005 -0634 00 4 02626    STABL SXD IR4,4
00006  0074 00 4 00000          CALL PTLVL
00007  1 00000 0 00011
00010  0 00006 0 00003
                          *THIS IS THE UNI GENERATOR
00011  0500 00 0 77461          CLA ORDER
00012  0622 00 0 00030          STD TXLL
00013  0774 00 1 00001          AXT 1,1
00014  0774 00 2 00001          AXT 1,2
00015 -0634 00 2 05301          SXD X2,2
00016  0774 00 4 00001          AXT 1,4
00017 -0500 00 4 77170    CALU  CAL UNIT,4
00020  0602 00 1 04673          SLW UNI,1
00021  0500 00 0 05301          CLA X2
00022  0622 00 1 05300          STD UNI2,1
00023  1 00001 1 00024          TXI *+1,1,1
00024  1 00001 4 00025          TXI *+1,4,1
00025 -3 00044 4 00017          TXL CALU,4,36
00026  1 00001 2 00027          TXI *+1,2,1
00027 -0634 00 2 05301          SXD X2,2
00030 -3 00000 1 00016    TXLL  TXL CALU-1,1,**
                          *END OF UNI GENERATOR
                          *       PRELIMINARIES
00031 -0534 00 1 77457          LXD DAT,1
00032 -0500 00 1 76336          CAL DROWS,1
00033  0602 00 1 72252          SLW MROWS,1
00034  2 00001 1 00032          TIX *-2,1,1
00035  0500 00 0 05306          CLA =O777777777777
00036  0601 00 0 02613          STO CEXP
00037  0601 00 0 02177          STO BEXP
                          *NEXT SECTION COMPUTES ASQA BLOCKS
00040 -0534 00 1 77461          LXD ORDER,1
00041  0760 00 0 00000    STA   CLM
00042 -0754 00 1 00000          PXD 0,1
00043  0601 00 0 05302          STO XX
00044  0402 00 0 77444          SUB ONED
00045 -0765 00 0 00044          LGR 36
00046  0500 00 0 05302          CLA XX
00047  0767 00 0 00012          ALS 10
00050  0601 00 0 03652          STO SUB
00051  0500 00 0 05303          CLA =0
00052  0200 00 0 05302          MPY XX
00053  0767 00 0 00021          ALS 17
00054  0400 00 0 77444          ADD ONED
00055  0771 00 0 00001          ARS 1
```

```
00056  0601 00 0 02617          STO DTOFP
00057 -0625 00 0 00542          STL TRAP
00060  0020 00 0 00512          TRA FPG
00061  0500 00 0 02623          CLA FP
00062  0601 00 1 01571          STO ASQA,1
00063  0402 00 0 03652          SUB SUB
00064  0120 00 0 00067          TPL *+3
00065  0600 00 1 02176          STZ ASQA2,1
00066  0020 00 0 00070          TRA TIXS
00067  0601 00 1 02176          STO ASQA2,1
00070  2 00001 1 00041    TIXS  TIX STA,1,1
                         *    COMPUTE LUM
00071 -0534 00 1 77457    CC    LXD DAT,1
00072  0600 00 0 04266          STZ TOTAL
00073  0500 00 0 04266          CLA TOTAL
00074  0560 00 1 72252          LDQ MROWS,1
00075  0522 60 0 00001          XEC* $COUNT
00076  0622 00 0 04266          STD TOTAL
00077  2 00001 1 00073          TIX *-4,1,1
00100  0500 00 0 04266          CLA TOTAL
00101  0771 00 0 00001          ARS 1
00102  0622 00 0 04266          STD TOTAL
00103  0601 00 0 02617          STO DTOFP
00104 -0625 00 0 00542          STL     TRAP
00105  0020 00 0 00512          TRA     FPG
00106  0500 00 0 02623          CLA     FP
00107 -0534 00 1 77461          LXD ORDER,1
00110  0560 00 0 05303          LDQ =0
00111  0240 00 1 01571          FDH ASQA,1
00112 -0600 00 0 02622          STQ FLUM
                         * NEXT SECTION COMPUTES ZERO VALUES
                         * FOR TOP AND BOT
00113  0500 00 0 02175          CLA ASQA2-1
00114  0601 00 0 02611          STO BOTO
00115  0500 00 0 01570          CLA ASQA-1
00116 -0760 00 0 00003          SSM
00117  0601 00 0 04264          STO TOPO
                         * NEXT SECTION GENERATES INDIRECT
                         * ADDRESSING BEHIND SETS AS IN GENER
00120  0500 00 0 77461          CLA ORDER
00121  0622 00 0 03240          STD S902
00122  0500 00 0 00141    FF    CLA MXM1
00123  0602 00 0 03644          SLW SETS-1
00124  0500 00 0 77461          CLA ORDER
00125  0622 00 0 00137          STD LOC
00126  0500 00 0 77460          CLA NWORD
00127  0400 00 0 77444          ADD ONED
00130  0771 00 0 00022          ARS 18
00131  0601 00 0 00142          STO DIFSP
00132  0774 00 1 00001          AXT 1,1
00133  0500 00 1 03645          CLA SETS,1
00134  0402 00 0 00142          SUB DIFSP
00135  1 00001 1 00136          TXI *+1,1,1
00136  0602 00 1 03645          SLW SETS,1
00137 -3 00000 1 00134    LOC   TXL *-3,1,**
00140  0020 00 0 00143          TRA *+3
00141  000000247616       MXM1  VFD 18/0,03/2,15/MACRO
00142  0 00000 0 00000    DIFSP PZE
```

```
00143  0761 00 0 00000          NOP
                          * NEXT SECTION PUTS ONE ELEMENT SETS
                          *    IN NUMERICAL ORDER BEHIND SETS
                          *    AND SETS ONE ELEMENT VALUES FOR
                          *    BOT, TOP, AA AND LA
00144 -0534 00 1 77461    START LXD ORDER,1
00145  0500 00 1 05300          CLA UNI2,1
00146 -0734 00 2 00000          PDX 0,2
00147 -0500 00 1 04673          CAL UNI,1
00150  0602 60 1 03645          SLW* SETS,1
00151  2 00001 1 00145          TIX *-4,1,1
00152 -0534 00 1 77461          LXD ORDER,1
00153  0500 00 0 77444          CLA ONED
00154  0622 00 1 01164          STD AA,1
00155  0600 00 1 03233          STZ LA,1
00156  2 00001 1 00154          TIX *-2,1,1
00157 -0534 00 1 77461          LXD ORDER,1
00160  0500 00 0 04264          CLA TOPO
00161  0601 00 1 04263          STO TOP,1
00162  2 00001 1 00161          TIX *-1,1,1
00163 -0534 00 1 77461          LXD ORDER,1
00164  0500 00 0 02611          CLA BOTO
00165  0601 00 1 02610          STO BOT,1
00166  2 00001 1 00165          TIX *-1,1,1
                          *NOW START HILL CLIMB
                          * NEXT SECTION PICKS OUT SET WHICH
                          *    HAS NEXT ELEMENT IN IT
00167 -0534 00 1 77461    HILL  LXD ORDER,1
00170 -0534 00 4 77461    ONE   LXD ORDER,4
00171  0500 00 1 05300          CLA UNI2,1
00172 -0734 00 2 00000          PDX 0,2
00173 -0500 00 1 04673          CAL UNI,1
00174 -0320 60 4 03645          ANA* SETS,4
00175 -0100 00 0 00200          TNZ *+3
00176  2 00001 4 00173          TIX *-3,4,1
00177  0000 00 0 00000          HTR
00200  0560 60 4 03645          LDQ* SETS,4
00201 -0600 00 0 03647          STQ SETSO
00202 -0634 00 1 02621          SXD ELMNT,1
00203  0322 60 4 03645          ERA* SETS,4
00204  0602 60 4 03645          SLW* SETS,4
00205  0500 00 4 02610          CLA BOT,4
00206  0601 00 0 02203          STO BOUT
00207  0500 00 4 04263          CLA TOP,4
00210  0601 00 0 03656          STO TOUT
00211 -0634 00 4 03651          SXD SOUT,4
                          *NEXT SECTION DEFINES NEW BOT FOR SOUT
00212  0500 00 4 01164    VALO  CLA AA,4
00213  0402 00 0 77444          SUB ONED
00214 -0734 00 4 00000          PDX 0,4
00215  0601 00 0 03652          STO SUB
00216  0500 00 4 02176          CLA ASQA2,4
00217 -0534 00 4 03651          LXD SOUT,4
00220  0601 00 4 02610          STO BOT,4
                          *NEXT SECTION DEFINES NEW TOP FOR SOUT
00221 -0534 00 2 77460          LXD NWORD,2
00222  0600 00 0 03237          STZ RL
00223 -0500 60 1 76742    OCNT  CAL* MATAX,1
```

```
00224 -0320 60 4 03645          ANA* SETS,4
00225 -0765 00 0 00044          LGR 36
00226  0500 00 0 03237          CLA RL
00227  0522 60 0 00001          XEC* $COUNT
00230  0622 00 0 03237          STD RL
00231  2 00001 2 00223          TIX OCNT,2,1
00232  0500 00 4 03233          CLA LA,4
00233 -0634 00 4 03651          SXD SOUT,4
00234 -0534 00 4 03652          LXD SUB,4
00235  0402 00 0 03237          SUB RL
00236  0601 00 0 02617          STO DTOFP
00237 -0625 00 0 00542          STL TRAP
00240  0020 00 0 00512          TRA FPG
00241  0500 00 0 02623          CLA FP
00242  0240 00 0 02622          FDH FLUM
00243  0131 00 0 00000          XCA
00244  0302 00 4 01571          FSB ASQA,4
00245 -0534 00 4 03651          LXD SOUT,4
00246  0601 00 4 04263          STO TOP,4
                         * NEXT SECTION TRIES ADDING CHOSEN
                         *    ELEMENT TO VARIOUS NEW SETS
00247 -0534 00 4 77461          LXD ORDER,4
00250 -0634 00 4 03650    FOUR  SXD SIN,4
00251  0500 00 0 03651          CLA SOUT
00252  0340 00 0 03650          CAS SIN
00253  0020 00 0 00256          TRA *+3
00254  0020 00 0 00406          TRA SKIP
00255  0020 00 0 00256          TRA *+1
00256  0500 00 4 02610          CLA BOT,4
00257  0601 00 0 02201          STO BIN
00260  0500 00 4 04263          CLA TOP,4
00261  0601 00 0 03654          STO TIN
00262 -0634 00 4 03650          SXD SIN,4
                         *NEXT SECTION DEFINES NEW BOT FOR SIN
00263  0500 00 4 01164    VALI  CLA AA,4
00264  0400 00 0 77444          ADD ONED
00265 -0734 00 4 00000          PDX 0,4
00266  0601 00 0 03652          STO SUB
00267  0500 00 4 02176          CLA ASQA2,4
00270 -0534 00 4 03650          LXD SIN,4
00271  0601 00 4 02610          STO BOT,4
                         *NEXT SECTION DEFINES NEW TOP FOR SIN
00272 -0534 00 2 77460          LXD NWORD,2
00273  0600 00 0 03237          STZ RL
00274 -0500 60 1 76742    ICNT  CAL* MATAX,1
00275 -0320 60 4 03645          ANA* SETS,4
00276 -0765 00 0 00044          LGR 36
00277  0500 00 0 03237          CLA RL
00300  0522 60 0 00001          XEC* $COUNT
00301  0622 00 0 03237          STD RL
00302  2 00001 2 00274          TIX ICNT,2,1
00303  0500 00 4 03233          CLA LA,4
00304 -0634 00 4 03650          SXD SIN,4
00305 -0534 00 4 03652          LXD SUB,4
00306  0400 00 0 03237          ADD RL
00307  0601 00 0 02617          STO DTOFP
00310 -0625 00 0 00542          STL TRAP
00311  0020 00 0 00512          TRA FPG
```

```
00312  0500 00 0 02623          CLA FP
00313  0240 00 0 02622          FDH FLUM
00314  0131 00 0 00000          XCA
00315  0302 00 4 01571          FSB ASQA,4
00316 -0534 00 4 03650          LXD SIN,4
00317  0601 00 4 04263          STO TOP,4
00320  0500 00 1 05300          CLA UNI2,1
00321 -0734 00 2 00000          PDX 0,2
00322  0560 60 4 03645          LDQ* SETS,4
00323 -0600 00 0 03646          STQ SETSI
00324 -0500 00 1 04673          CAL UNI,1
00325 -0602 60 4 03645          ORS* SETS,4
                         *NEXT SECTION COMPUTES VALUE OF EXP
00326 -0534 00 1 77461    COMPU LXD ORDER,1
00327  0600 00 0 04265          STZ TOPP
00330  0600 00 0 02612          STZ BOTT
00331  0500 00 0 04265          CLA TOPP
00332  0300 00 1 04263          FAD TOP,1
00333  0601 00 0 04265          STO TOPP
00334  0500 00 0 02612          CLA BOTT
00335  0300 00 1 02610          FAD BOT,1
00336  0601 00 0 02612          STO BOTT
00337  2 00001 1 00331          TIX *-6,1,1
00340 -0534 00 1 02621          LXD ELMNT,1
00341  0500 00 0 04265          CLA TOPP
00342  0120 00 0 00344          TPL *+2
00343  0760 00 0 00002          CHS
00344  0131 00 0 00000          XCA
00345  0260 00 0 04265          FMP TOPP
00346  0560 00 0 05303          LDQ =0
00347  0240 00 0 02612          FDH BOTT
00350 -0600 00 0 02620          STQ EXP
                         * NEXT SECTION COMPARES EXP WITH CEXP
                         *   AND REPLACES IT IF BETTER
00351  0500 00 0 02620    MMM   CLA EXP
00352  0340 00 0 02613          CAS CEXP
00353  0020 00 0 00356          TRA *+3
00354  0020 00 0 00377          TRA NEXT
00355  0020 00 0 00377          TRA NEXT
00356  0601 00 0 02613          STO CEXP
00357  0500 00 0 02621          CLA ELMNT
00360  0601 00 0 02614          STO CE
00361  0500 00 0 03651          CLA SOUT
00362  0601 00 0 02616          STO CO
00363  0500 00 0 03650          CLA SIN
00364  0601 00 0 02615          STO CI
00365 -0534 00 4 03650          LXD SIN,4
00366  0500 00 4 04263          CLA TOP,4
00367  0601 00 0 03653          STO TI
00370  0500 00 4 02610          CLA BOT,4
00371  0601 00 0 02200          STO BI
00372 -0534 00 4 03651          LXD SOUT,4
00373  0500 00 4 04263          CLA TOP,4
00374  0601 00 0 03655          STO TO
00375  0500 00 4 02610          CLA BOT,4
00376  0601 00 0 02202          STO BO
                         * NEXT SECTION PUTS SETS BACK TO NORMAL
                         *   TO TRY NEXT RELOCATION OF ELEMENT
```

```
00377 -0534 00 4 03650   NEXT  LXD SIN,4
00400 -0500 00 0 03646         CAL SETSI
00401  0602 60 4 03645         SLW* SETS,4
00402  0500 00 0 03654         CLA TIN
00403  0601 00 4 04263         STO TOP,4
00404  0500 00 0 02201         CLA BIN
00405  0601 00 4 02610         STO BOT,4
00406  2 00001 4 00250   SKIP  TIX FOUR,4,1
00407 -0534 00 4 03651         LXD SOUT,4
00410 -0500 00 0 03647         CAL SETSO
00411  0602 60 4 03645         SLW* SETS,4
00412  0500 00 0 03656         CLA TOUT
00413  0601 00 4 04263         STO TOP,4
00414  0500 00 0 02203         CLA BOUT
00415  0601 00 4 02610         STO BOT,4
00416  2 00001 1 00170         TIX ONE,1,1
                         * NEXT SECTION TESTS TO SEE WHETHER
                         *   THE LAST CYCLE OF THE HILL CLIMB
                         *   HAS IMPROVED THE DECOMPOSITION,
                         *   AND REPLACES IF THE ANSWER IS YES
00417  0500 00 0 02177         CLA BEXP
00420  0340 00 0 02613         CAS CEXP
00421  0000 00 0 00000         HTR
00422  0020 00 0 00543         TRA PRINT
00423  0500 00 0 02613   REPLA CLA CEXP
00424  0601 00 0 02177         STO BEXP
00425 -0534 00 1 02614         LXD CE,1
00426 -0534 00 4 02616         LXD CO,4
00427  0500 00 1 05300         CLA UNI2,1
00430 -0734 00 2 00000         PDX 0,2
00431 -0500 00 1 04673         CAL UNI,1
00432  0322 60 4 03645         ERA* SETS,4
00433  0602 60 4 03645         SLW* SETS,4
00434  0500 00 4 01164         CLA AA,4
00435  0402 00 0 77444         SUB ONED
00436  0601 00 4 01164         STO AA,4
00437 -0534 00 2 77460         LXD NWORD,2
00440  0600 00 0 03237         STZ RL
00441 -0500 60 1 76742         CAL* MATAX,1
00442 -0320 60 4 03645         ANA* SETS,4
00443 -0765 00 0 00044         LGR 36
00444  0500 00 0 03237         CLA RL
00445  0522 60 0 00001         XEC* $COUNT
00446  0622 00 0 03237         STD RL
00447  2 00001 2 00441         TIX *-6,2,1
00450  0500 00 4 03233         CLA LA,4
00451  0402 00 0 03237         SUB RL
00452  0601 00 4 03233         STO LA,4
00453  0500 00 0 03655         CLA TO
00454  0601 00 4 04263         STO TOP,4
00455  0500 00 0 02202         CLA BO
00456  0601 00 4 02610         STO BOT,4
00457 -0534 00 4 02615         LXD CI,4
00460  0500 00 1 05300         CLA UNI2,1
00461 -0734 00 2 00000         PDX 0,2
00462 -0500 00 1 04673         CAL UNI,1
00463 -0602 60 4 03645         ORS* SETS,4
00464  0500 00 4 01164         CLA AA,4
```

```
00465  0400 00 0 77444          ADD ONED
00466  0601 00 4 01164          STO AA,4
00467 -0534 00 2 77460          LXD NWORD,2
00470  0600 00 0 03237          STZ RL
00471 -0500 60 1 76742          CAL* MATAX,1
00472 -0320 60 4 03645          ANA* SETS,4
00473 -0765 00 0 00044          LGR 36
00474  0500 00 0 03237          CLA RL
00475  0522 60 0 00001          XEC* $COUNT
00476  0622 00 0 03237          STD RL
00477  2 00001 2 00471          TIX *-6,2,1
00500  0500 00 4 03233          CLA LA,4
00501  0400 00 0 03237          ADD RL
00502  0601 00 4 03233          STO LA,4
00503  0500 00 0 03653          CLA TI
00504  0601 00 4 04263          STO TOP,4
00505  0500 00 0 02200          CLA BI
00506  0601 00 4 02610          STO BOT,4
00507  0020 00 0 00167          TRA HILL
00510 -0534 00 4 02626   OUT    LXD IR4,4
00511  0020 00 4 00001          TRA 1,4
                         * SUBROUTINE FOR GENERATING FLOATING
                         *  POINT VERSION OF DECREMENT INTEGERS
00512 -0500 00 0 00542   FPG    CAL       TRAP
00513  0400 00 0 05304          ADD       =1
00514  0602 00 0 00542          SLW       TRAP
00515 -0634 00 2 03236          SXD       R2,2
00516  0774 00 2 00001          AXT       1,2
00517  0500 00 0 02617          CLA       DTOFP
00520 -0100 00 0 00523          TNZ ALS
00521  0600 00 0 02623          STZ FP
00522  0020 00 0 00541          TRA TRAP-1
00523  0767 00 0 00001   ALS    ALS       1
00524 -0760 00 0 00001          PBT
00525  1 00001 2 00523          TXI       ALS,2,1
00526  0771 00 0 00011          ARS       9
00527  0601 00 0 02623          STO       FP
00530  0754 00 2 00000          PXA       0,2
00531  0402 00 0 05305          SUB       =146
00532  0767 00 0 00033          ALS       27
00533  0760 00 0 00003          SSP
00534  0400 00 0 02623          ADD       FP
00535  0601 00 0 02623          STO       FP
00536  0760 00 0 00000          CLM
00537  0300 00 0 02623          FAD FP
00540  0601 00 0 02623          STO FP
00541 -0534 00 2 03236          LXD       R2,2
00542  0020 00 0 00000   TRAP   TRA       **
                         * NEXT SECTION PRINTS OUT COMPLETE
                         *   DECOMPOSITION AT END OF HILL CLIMB
00543 -0534 00 1 77461   PRINT  LXD ORDER,1
00544 -0534 00 2 77460          LXD NWORD,2
00545 -0500 60 1 03645          CAL* SETS,1
00546  0602 00 2 77416          SLW SET,2
00547  2 00001 2 00545          TIX *-2,2,1
00550 -0634 00 1 02625          SXD IR1,1
00551  0074 00 4 00002          CALL PTSET
00552  1 00000 0 00554
```

```
00553   0 01047 0 00003
00554  -0534 00 1 02625              LXD IR1,1
00555   2 00001 1 00544              TIX PRINT+1,1,1
00556   0020 00 0 00510              TRA OUT
00557   0 00000 0 00000              PZE
01164                                BES 260
01164   0 00000 0 00000    AA
01571                                BES 260
01571   0 00000 0 00000    ASQA
02176                                BES 260
02176   0 00000 0 00000    ASQA2
02177   0 00000 0 00000    BEXP
02200   0 00000 0 00000    BI
02201   0 00000 0 00000    BIN
02202   0 00000 0 00000    BO
02203   0 00000 0 00000    BOUT
02610                                BES 260
02610   0 00000 0 00000    BOT
02611   0 00000 0 00000    BOTO
02612   0 00000 0 00000    BOTT
02613   0 00000 0 00000    CEXP
02614   0 00000 0 00000    CE
02615   0 00000 0 00000    CI
02616   0 00000 0 00000    CO
02617   0 00000 0 00000    DTOFP
02620   0 00000 0 00000    EXP
02621   0 00000 0 00000    ELMNT
02622   0 00000 0 00000    FLUM
02623   0 00000 0 00000    FP
02624   0 00000 0 00000    HOLD
02625   0 00000 0 00000    IR1
02626   0 00000 0 00000    IR4
03233                                BES 260
03233   0 00000 0 00000    LA
03234   0 00000 0 00000    LUM
03235   0 00000 0 00000    RI
03236   0 00000 0 00000    R2
03237   0 00000 0 00000    RL
03240   0 00000 0 00000    S902
03645                                BES 260
03645   0 00000 0 00000    SETS
03646   0 00000 0 00000    SETSI
03647   0 00000 0 00000    SETSO
03650   0 00000 0 00000    SIN
03651   0 00000 0 00000    SOUT
03652   0 00000 0 00000    SUB
03653   0 00000 0 00000    TI
03654   0 00000 0 00000    TIN
03655   0 00000 0 00000    TO
03656   0 00000 0 00000    TOUT
04263                                BES 260
04263   0 00000 0 00000    TOP
04264   0 00000 0 00000    TOPO
04265   0 00000 0 00000    TOPP
04266   0 00000 0 00000    TOTAL
04673                                BES 260
04673   0 00000 0 00000    UNI
05300                                BES 260
```

```
05300   0 00000 0 00000    UNI2
05301   0 00000 0 00000    X2
05302   0 00000 0 00000    XX
                         *    COMMON BLOCK AS IN HIDECS 2
                77462            COMMON -1
                77462      INDIC COMMON 1
                77461      ORDER COMMON 1
                77460      NWORD COMMON 1
                77457      DAT   COMMON 1
                77456      LGTH  COMMON 1
                77455      LATIS COMMON 1
                77454      NBITH COMMON 1
                77453      NBITL COMMON 1
                77452      NBIT1 COMMON 1
                77451      NBIT  COMMON 1
                77450      NSQ1  COMMON 1
                77447      OPRMN COMMON 1
                77446      ATOMO COMMON 1
                77445      ATOM  COMMON 1
                77444      ONED  COMMON 1
                77443      D36   COMMON 1
                77442      ATOOX COMMON 10
                77430      ATOX  COMMON 10
                77416      SET   COMMON 10
                77404      RANDM COMMON 10
                77372      DIFF  COMMON 10
                77360      CONVT COMMON 40
                77310      DATA  COMMON 40
                77240      MATA  COMMON 40
                77170      UNIT  COMMON 40
                77120      COMUN COMMON 40
                77050      EQLS  COMMON 20
                77024      SECTS COMMON 50
                76742      MATAX COMMON 260
                76336      DROWS COMMON 2100
                72252      MROWS COMMON 2100
                66166      INMAT COMMON 5400
                53536      ATMS  COMMON 2000
                47616      MACRO COMMON 7000
                                 END

  LITERALS
05303   000000000000
05304   000000000001
05305   000000000222
05306   777777777777

05307 IS THE FIRST LOCATION NOT USED BY THIS PROGRAM


   NO ERROR IN ABOVE ASSEMBLY
* DATE AND TIME NOW      32/67 5498.1
```

The decomposition of the sample graph by STABL, follows:

```
 0
14  15  16  17
 0
 0
 0
 0
11  12  13
 0
 6   7   8   9  10
 0
 0
 0
 0
 1   2   3   4   5
 0
 0
 0
```

SIMPX

The best way to visualize SIMPX is as follows.

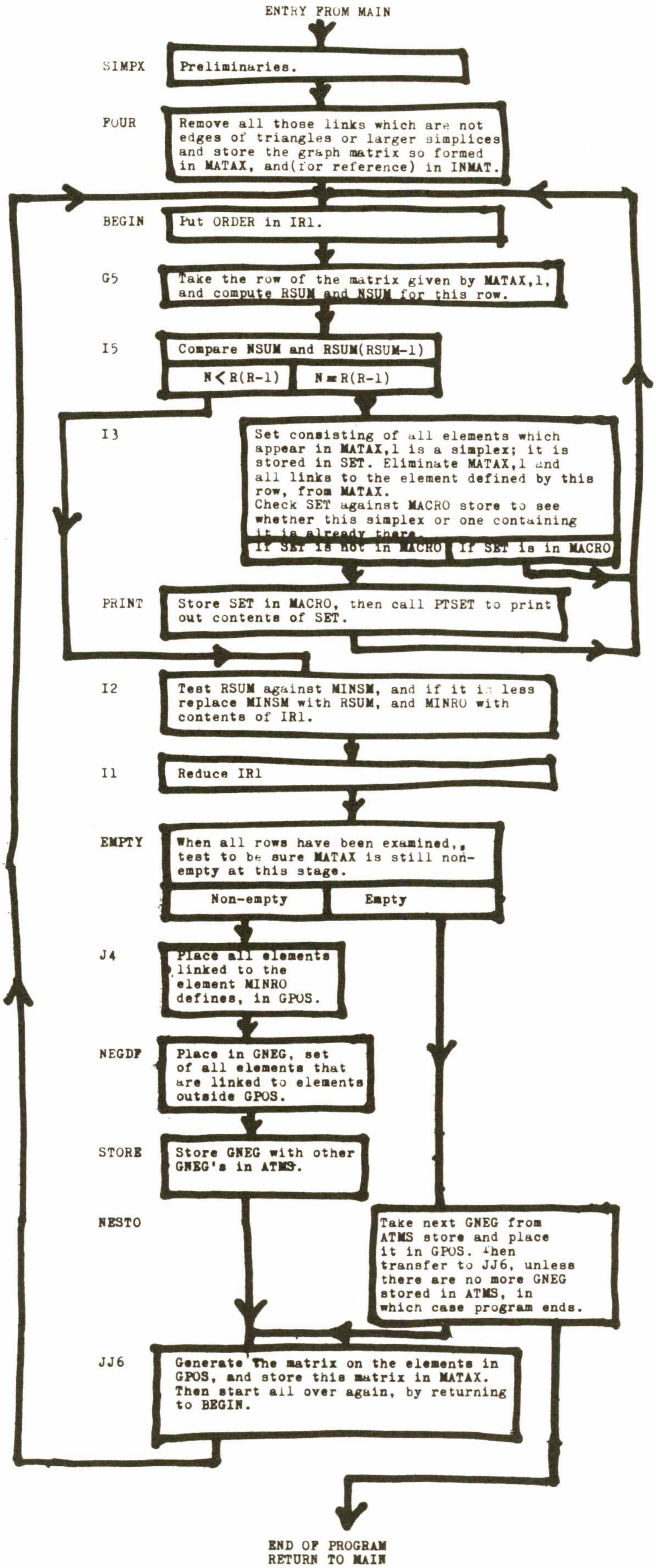
A graph, since it consists only of vertices (elements) and edges (links), is of course a topological 1-complex. However, we may very easily represent any graph as a many-dimensional complex as follows. A set of vertices has the property that each two vertices in the set are connected by an edge of G, is often called a complete subgraph. Replace each complete subgraph of G by the simplex generated by its vertices. This means that for three points all connected to one another, replace them by a triangle, for four such points, replace with a tetrahedron. Clearly the vertices of the resulting topological complex are precisely the elements of G, and its edges are the links of G.

A most natural decomposition of G, into subsystems, once we have abandoned the idea of trying to decompose it into discrete subsystems, is the decomposition in which each maximal simplex of our topological complex defines a subsystem - namely the set of its vertices.

In this case the decomposition presents no intuitive difficulty - in fact, the simplices of G can easily be picked out from the graph by inspection. However, since the task of enumerating all the simplices is rather laborious, it is useful to have it done mechanically. The program SIMPX is based on an algorithm given by Harary and Ross (1957). This algorithm uses a theorem,which gives a necessary and sufficient condition on the graph matrix, for a vertex to belong to just one simplex. This theorem is only applicable when every simplex has at least 3 vertices. The program therefore begins by removing the isolated links of G.

The algorithm then tests one element at a time, until it finds one which satisfies the condition; when it finds such an element, the program removes it from G, at the same time printing out the unique simplex this vertex defines. Then the program starts again applying the same algorithm to the reduced G.

If no vertex of G is in just one simplex, the graph G is split into two simplex-disjoint subgraphs, and the algorithm applied to these subgraphs in turn.


ENTRY FROM MAIN
SIMPX
Preliminaries.
FOUR
Remove all those links which are not edges of triangles or larger simplices and store the graph matrix so formed in MATAX, and(for reference) in INMAT.
BEGIN
Put ORDER in IR1.
G5
Take the row of the matrix given by MATAX,1, and compute RSUM and NSUM for this row.
I5
Compare NSUM and RSUM(RSUM-1)
N < R(R-1)
N = R(R-1)
I3
Set consisting of all elements which appear in MATAX,1 is a simplex; it is stored in SET. Eliminate MATAX,1 and all links to the element defined by this row, from MATAX.
Check SET against MACRO store to see whether this simplex or one containing it is already there.
If SET is not in MACRO
If SET is in MACRO
PRINT
Store SET in MACRO, then call PTSET to print out contents of SET.
I2
Test RSUM against MINSM, and if it is less replace MINSM with RSUM, and MINRO with contents of IR1.
I1
Reduce IR1
EMPTY
When all rows have been examined,, test to be sure MATAX is still non-empty at this stage.
Non-empty
Empty
J4
Place all elements linked to the element MINRO defines, in GPOS.
NEGDF
Place in GNEG, set of all elements that are linked to elements outside GPOS.
STORE
Store GNEG with other GNEG's in ATMS.
NESTO
Take next GNEG from ATMS store and place it in GPOS. Then transfer to JJ6, unless there are no more GNEG stored in ATMS, in which case program ends.
JJ6
Generate the matrix on the elements in GPOS, and store this matrix in MATAX. Then start all over again, by returning to BEGIN.
END OF PROGRAM
RETURN TO MAIN

```
*     FAP

                    00004          ENTRY   SIMPX


  TRANSFER VECTOR
00000  234664456360     COUNT
00001  476362256360     PTSET

  LINKAGE DIRECTOR
00002  000000000000
00003  623144476760

00004 -0634 00 4 00352   SIMPX SXD IR4,4
                        *THIS IS THE UNI GENERATOR
00005  0500 00 0 77461         CLA ORDER
00006  0622 00 0 00024         STD TXLL
00007  0774 00 1 00001         AXT 1,1
00010  0774 00 2 00001         AXT 1,2
00011 -0634 00 2 02404         SXD X2,2
00012  0774 00 4 00001         AXT 1,4
00013 -0500 00 4 77170   CALU  CAL UNIT,4
00014  0602 00 1 01776         SLW UNI,1
00015  0500 00 0 02404         CLA X2
00016  0622 00 1 02403         STD UNI2,1
00017  1 00001 1 00020         TXI *+1,1,1
00020  1 00001 4 00021         TXI *+1,4,1
00021 -3 00044 4 00013         TXL CALU,4,36
00022  1 00001 2 00023         TXI *+1,2,1
00023 -0634 00 2 02404         SXD X2,2
00024 -3 00000 1 00012   TXLL  TXL CALU-1,1,**
                        * NEXT SECTION ERADICATES ALL PAIRS
                        *    FROM MATAX
00025 -0534 00 1 77461         LXD ORDER,1
00026 -0534 00 4 77461         LXD ORDER,4
00027 -0534 00 2 77460   FOUR  LXD NWORD,2
00030  0600 00 0 01371         STZ RINC
00031 -0500 60 1 76742   CIRC  CAL* MATAX,1
00032 -0320 60 4 76742         ANA* MATAX,4
00033 -0765 00 0 00044         LGR 36
00034  0500 00 0 01371         CLA RINC
00035  0522 60 0 00000         XEC* $COUNT
00036  0622 00 0 01371         STD RINC
00037  2 00001 2 00031         TIX CIRC,2,1
00040  0500 00 0 01371         CLA RINC
00041 -0100 00 0 00047         TNZ NEX
00042  0500 00 4 02403         CLA UNI2,4
00043 -0734 00 2 00000         PDX 0,2
00044 -0500 00 4 01776         CAL UNI,4
00045  0760 00 0 00006         COM
00046  0320 60 1 76742         ANS* MATAX,1
00047  2 00001 4 00027   NEX   TIX FOUR,4,1
00050  2 00001 1 00026         TIX FOUR-1,1,1
                        * NEXT SECTION PUTS PERMANENT REFERENCE
                        *    MATRIX,WITHOUT PAIRS) IN INMAT
00051 -0534 00 2 77457         LXD DAT,2
00052 -0500 00 2 72252         CAL MROWS,2
00053  0602 00 2 66166         SLW INMAT,2
```

```
00054   2 00001 2 00052          TIX *-2,2,1
                               * NEXT SECTION TAKES NEXT NON-ZERO
                               * ROW AND COMPUTES ITS RSUM AND NSUM
00055 -0534 00 1 77461          BEGIN LXD ORDER,1
00056  0500 00 0 02405                CLA =0377777777777
00057  0601 00 0 00353                STO MINSM
00060  0760 00 0 00000          G5    CLM
00061 -0534 00 2 77460                LXD NWORD,2
00062 -0501 60 1 76742                ORA* MATAX,1
00063   2 00001 2 00062               TIX *-1,2,1
00064 -0100 00 0 00066                TNZ *+2
00065   2 00001 1 00060               TIX G5,1,1
00066 -0534 00 4 77461                LXD ORDER,4
00067  0600 00 1 00762                STZ NSUM,1
00070  0600 00 1 01370                STZ RSUM,1
00071  0500 00 4 02403          G4    CLA UNI2,4
00072 -0734 00 2 00000                PDX 0,2
00073 -0500 00 4 01776                CAL UNI,4
00074 -0320 60 1 76742                ANA* MATAX,1
00075  0100 00 0 00112                TZE TIXG4
00076 -0534 00 2 77460                LXD NWORD,2
00077  0600 00 0 01371                STZ RINC
00100 -0500 60 1 76742                CAL* MATAX,1
00101 -0320 60 4 76742                ANA* MATAX,4
00102 -0765 00 0 00044                LGR 36
00103  0500 00 0 01371                CLA RINC
00104  0522 60 0 00000                XEC* $COUNT
00105  0622 00 0 01371                STD RINC
00106   2 00001 2 00100               TIX *-6,2,1
00107  0500 00 0 01371                CLA RINC
00110  0400 00 1 01370                ADD RSUM,1
00111  0601 00 1 01370                STO RSUM,1
00112   2 00001 4 00071         TIXG4 TIX G4,4,1
00113 -0534 00 2 77460                LXD NWORD,2
00114  0560 60 1 76742          G2    LDQ* MATAX,1
00115  0500 00 1 00762                CLA NSUM,1
00116  0522 60 0 00000                XEC* $COUNT
00117  0622 00 1 00762                STD NSUM,1
00120   2 00001 2 00114               TIX G2,2,1
                               * NEXT SECTION TESTS NSUM AND RSUM
                               *    INEQUALITY
00121  0500 00 1 00762          I5    CLA NSUM,1
00122  0100 00 0 00210                TZE I1
00123  0560 00 1 00762                LDQ NSUM,1
00124  0200 00 1 00762                MPY NSUM,1
00125  0767 00 0 00021                ALS 17
00126  0402 00 1 00762                SUB NSUM,1
00127  0340 00 1 01370                CAS RSUM,1
00130  0020 00 0 00202                TRA I2
00131  0020 00 0 00133                TRA I3
A 00132  0000 00 0 00000              HTR
00133 -0534 00 2 77460          I3    LXD NWORD,2
00134 -0500 60 1 76742                CAL* MATAX,1
00135  0602 00 2 77416                SLW SET,2
00136  0600 60 1 76742                STZ* MATAX,1
00137   2 00001 2 00134               TIX *-3,2,1
```

```
00140  0500 00 1 02403          CLA UNI2,1
00141 -0734 00 2 00000          PDX 0,2
00142 -0500 00 1 01776          CAL UNI,1
00143 -0602 00 2 77416          ORS SET,2
00144  0760 00 0 00006          COM
00145 -0534 00 4 77461          LXD ORDER,4
00146  0320 60 4 76742          ANS* MATAX,4
00147  2 00001 4 00146          TIX *-1,4,1
                         * NEXT SECTION TESTS TO SEE WHETHER
                         *    ANY SIMPLEX ALREADY  STORED IN MACRO
                         *    CONTAINS SET AS A SUBSET OF ITSELF
                         *    IF SO,  NO PRINT TAKES PLACE
00150 -0534 00 4 00763          LXD OUTIN,4
00151  0600 00 0 00323          STZ CHEK
00152 -0534 00 2 77460   LXN    LXD NWORD,2
00153 -0500 00 4 47616          CAL MACRO,4
00154 -0501 00 2 77416          ORA SET,2
00155  0322 00 4 47616          ERA MACRO,4
00156 -0602 00 0 00323          ORS CHEK
00157  2 00001 4 00160          TIX *+1,4,1
00160  2 00001 2 00153          TIX LXN+1,2,1
00161 -0500 00 0 00323          CAL CHEK
00162  0100 00 0 00055          TZE BEGIN
00163  2 00001 4 00151   PRINT  TIX LXN-1,4,1
00164  0500 00 0 00763          CLA OUTIN
00165  0400 00 0 77460          ADD NWORD
00166  0400 00 0 77444          ADD ONED
00167  0622 00 0 00763          STD OUTIN
00170 -0534 00 4 00763          LXD OUTIN,4
00171 -0534 00 2 77460          LXD NWORD,2
00172 -0500 00 2 77416          CAL SET,2
00173  0602 00 4 47616          SLW MACRO,4
00174  2 00001 4 00175          TIX *+1,4,1
00175  2 00001 2 00172          TIX *-3,2,1
00176  0 07400 4 00001          CALL PTSET
00177  1 00000 0 00201
00200  0 10206 0 00002
00201  0020 00 0 00055          TRA BEGIN
00202  0500 00 1 01370   I2     CLA RSUM,1
00203  0340 00 0 00353          CAS MINSM
00204  0020 00 0 00210          TRA *+4
00205  0020 00 0 00210          TRA *+3
00206  0601 00 0 00353          STO MINSM
00207 -0634 00 1 00354          SXD MINRO,1
00210  2 00001 1 00060   I1     TIX G5,1,1
00211 -0534 00 2 77457   EMPTY  LXD DAT,2
00212  0760 00 0 00000          CLM
00213 -0501 00 2 72252          ORA MROWS,2
00214  2 00001 2 00213          TIX *-1,2,1
00215  0100 00 0 00261          TZE NESTO
00216  0020 00 0 00217          TRA J4
                         * NEXT SECTION SEPARATES SET OF
                         *    ELEMENTS IN MINRO FROM THE REST
                         *    OF THE MATRIX
00217 -0534 00 1 00354   J4     LXD MINRO,1
00220 -0534 00 2 77460          LXD NWORD,2
```

```
00221 -0500 60 1 76742          CAL* MATAX,1
00222  0602 00 2 00336          SLW GPOS,2
00223  2 00001 2 00221          TIX *-2,2,1
00224  0500 00 1 02403          CLA UNI2,1
00225 -0734 00 2 00000          PDX 0,2
00226 -0500 00 1 01776          CAL UNI,1
00227 -0602 00 2 00336          ORS GPOS,2
                          * NEXT SECTION DEFINES GNEG AS LOGICAL
                          *   OR OF ALL ROWS OF THE MATRIX
                          *   WHOSE ELEMENTS DO NOT APPEAR IN
                          *   GPOS
00230 -0534 00 1 77461     NEGDF LXD ORDER,1
00231 -0534 00 2 77460          LXD NWORD,2
00232  0600 00 2 00351          STZ GNEG,2
00233  2 00001 2 00232          TIX *-1,2,1
00234  0500 00 1 02403     ONER  CLA UNI2,1
00235 -0734 00 2 00000          PDX 0,2
00236 -0500 00 2 00336          CAL GPOS,2
00237 -0320 00 1 01776          ANA UNI,1
00240 -0100 00 0 00245          TNZ DOWNO
00241 -0534 00 2 77460          LXD NWORD,2
00242 -0500 60 1 76742          CAL* MATAX,1
00243 -0602 00 2 00351          ORS GNEG,2
00244  2 00001 2 00242          TIX *-2,2,1
00245  2 00001 1 00234     DOWNO TIX ONER,1,1
                          *NEXT SECTION STORES GNEG IN  ATMS
00246  0500 00 0 00355     STORE CLA NEGIN
00247  0400 00 0 77460          ADD NWORD
00250  0400 00 0 77444          ADD ONED
00251  0622 00 0 00355          STD NEGIN
00252 -0534 00 4 00355          LXD NEGIN,4
00253 -0534 00 2 77460          LXD NWORD,2
00254 -0500 00 2 00351          CAL GNEG,2
00255  0602 00 4 53536          SLW ATMS,4
00256  2 00001 4 00257          TIX *+1,4,1
00257  2 00001 2 00254          TIX *-3,2,1
00260  0020 00 0 00273          TRA JJ6
                          * NEXT SECTION TAKES MOST RECENT
                          *   ADDITION TO ATMS BLOCK AND PUTS
                          *   IT IN GPOS
00261 -0534 00 4 00355     NESTO LXD NEGIN,4
00262 -3 00001 4 00320          TXL OUT,4,1
00263 -0534 00 2 77460          LXD NWORD,2
00264 -0500 00 4 53536          CAL ATMS,4
00265  0602 00 2 00336          SLW GPOS,2
00266  2 00001 4 00267          TIX *+1,4,1
00267  2 00001 2 00264          TIX *-3,2,1
00270  2 00001 4 00271          TIX *+1,4,1
00271 -0634 00 4 00355          SXD NEGIN,4
00272  0020 00 0 00273          TRA JJ6
                          * NEXT SECTION CREATES NEW MATRIX
                          *   FROM GPOS AND TRANSFERS  TO BEGIN
00273 -0534 00 2 77457     JJ6   LXD DAT,2
00274 -0500 00 2 66166          CAL INMAT,2
00275  0602 00 2 72252          SLW MROWS,2
00276  2 00001 2 00274          TIX *-2,2,1
```

```
00277 -0534 00 2 77460           LXD NWORD,2
00300 -0534 00 1 77461           LXD ORDER,1
00301 -0500 00 2 00336           CAL GPOS,2
00302  0320 60 1 76742           ANS* MATAX,1
00303  2 00001 1 00302           TIX *-1,1,1
00304  2 00001 2 00300           TIX *-4,2,1
00305 -0534 00 1 77461           LXD ORDER,1
00306  0500 00 1 02403    CLU    CLA UNI2,1
00307 -0734 00 2 00000           PDX 0,2
00310 -0500 00 2 00336           CAL GPOS,2
00311 -0320 00 1 01776           ANA UNI,1
00312 -0100 00 0 00316           TNZ SKIP
00313 -0534 00 2 77460           LXD NWORD,2
00314  0600 60 1 76742           STZ* MATAX,1
00315  2 00001 2 00314           TIX *-1,2,1
00316  2 00001 1 00306    SKIP   TIX CLU,1,1
00317  0020 00 0 00055           TRA BEGIN
00320 -0534 00 4 00352    OUT    LXD IR4,4
00321  0020 00 4 00001           TRA 1,4
00322  0 00000 0 00000           PZE
00323  0 00000 0 00000    CHEK   PZE
00336                            BES 10
00336  0 00000 0 00000    GPOS
00351                            BES 10
00351  0 00000 0 00000    GNEG
00352  0 00000 0 00000    IR4
00353  0 00000 0 00000    MINSM
00354  0 00000 0 00000    MINRO
00355  0 00000 0 00000    NEGIN
00762                            BES 260
00762  0 00000 0 00000    NSUM
00763  0 00000 0 00000    OUTIN  PZE
01370                            BES 260
01370  0 00000 0 00000    RSUM
01371  0 00000 0 00000    RINC
01776                            BES 260
01776  0 00000 0 00000    UNI
02403                            BES 260
02403  0 00000 0 00000    UNI2
02404  0 00000 0 00000    X2
                         *   COMMON BLOCK FROM HIDECS 2
                 77462           COMMON -1
                 77462    INDIC  COMMON 1
                 77461    ORDER  COMMON 1
                 77460    NWORD  COMMON 1
                 77457    DAT    COMMON 1
                 77456    LGTH   COMMON 1
                 77455    LATIS  COMMON 1
                 77454    NBITH  COMMON 1
                 77453    NBITL  COMMON 1
                 77452    NBIT1  COMMON 1
                 77451    NBIT   COMMON 1
                 77450    NSQ1   COMMON 1
                 77447    OPRMN  COMMON 1
                 77446    ATOMO  COMMON 1
                 77445    ATOM   COMMON 1
```

```
77444    ONED  COMMON 1
77443    D36   COMMON 1
77442    ATOOX COMMON 10
77430    ATOX  COMMON 10
77416    SET   COMMON 10
77404    RANDM COMMON 10
77372    DIFF  COMMON 10
77360    CONVT COMMON 40
77310    DATA  COMMON 40
77240    MATA  COMMON 40
77170    UNIT  COMMON 40
77120    COMUN COMMON 40
77050    EQLS  COMMON 20
77024    SECTS COMMON 50
76742    MATAX COMMON 260
76336    DROWS COMMON 2100
72252    MROWS COMMON 2100
66166    INMAT COMMON 5400
53536    ATMS  COMMON 2000
47616    MACRO COMMON 7000
               END

  LITERALS
02405   377777777777
```

The decomposition of the sample graph follows:

```
14 15 16 17
13 14 15
12 13 15
 7  9 10 11
 7  8  9 10
 6  7  8 10
 4  5  6
 2  4  5
 1  2  5
```

EQCLA

For many purposes the definition of subsystems used in SIMPX is rather restrictive. Thus for instance, suppose we have a set of five elements, connected to one another by 9 links, with just one pair of elements not linked directly. Clearly we should like to consider this set of five elements as a subsystem if it occurs in a larger more diffuse system. But SIMPX will define instead two four element subsystems, and not define the five element system at all.

A second example of a case where SIMPX is too rigid a criterion, is that where the links in the system are very sparse, so that perhaps there are not even any triangles; yet still some elements belong together more tightly than others; we need a weaker definition of subsystem than that used in SIMPX. One way of generalising SIMPX to make it more widely applicable, is given by Luce (1950). In EQCLA, we use another idea.

If we call two vertices x, y of G connected, whenever there exists a sequence of links (xa)(ab)(bc)...(dy), then it is a well known result of graph theory that connectedness is an equivalence relation, and hence that the elements of M fall into disjoint equivalence classes under this relation. These classes are usually called the components of the graph. Now, this is a very weak decomposition criterion; in other words, a completely connected graph has only one component, so the criterion offers us no way of telling anything about what its subsystems might be. It is therefore natural to try to generalise this notion of connectedness.

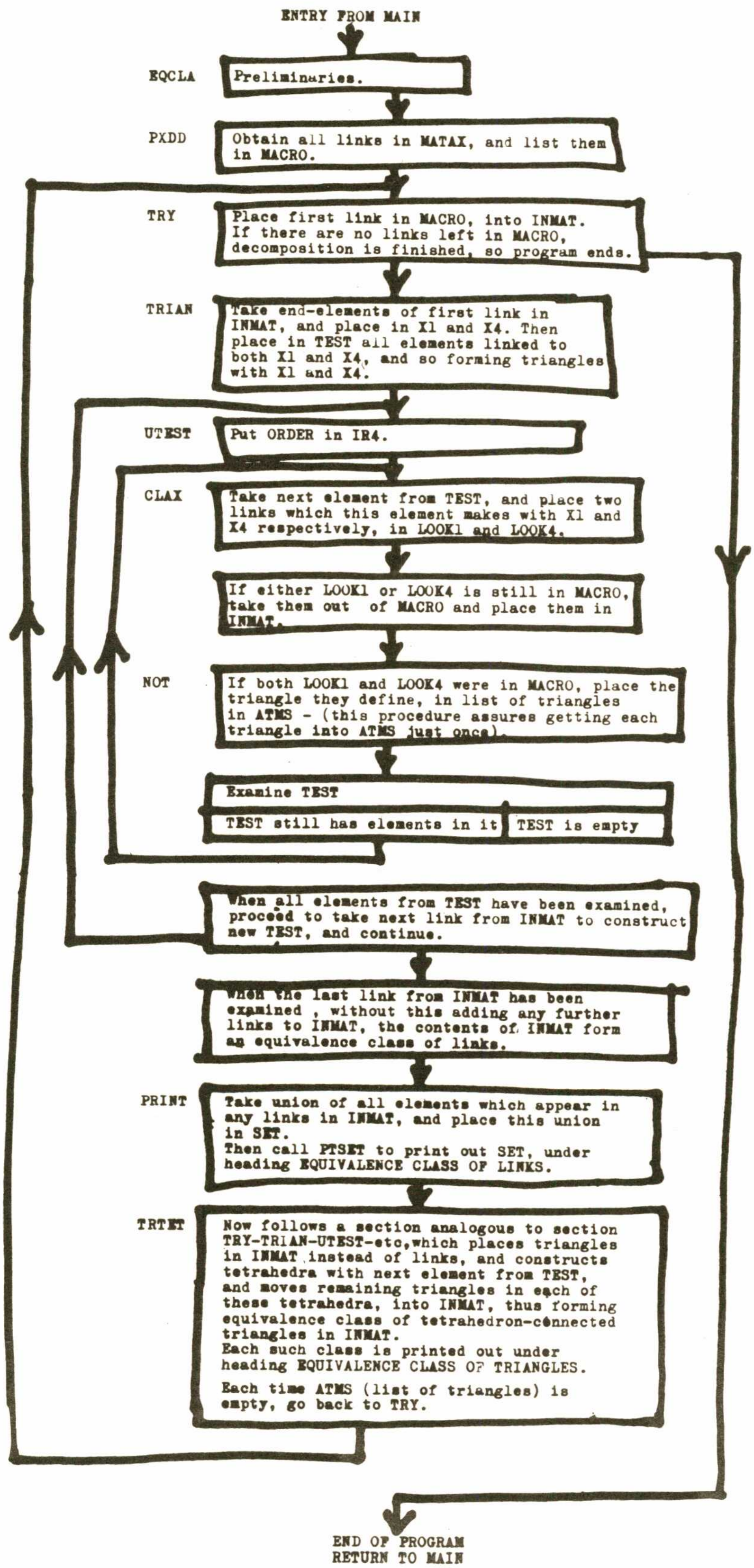
"Connectedness" is a relation between points, which we might call link-connectedness. Let us introduce a relation between links called triangle-connectedness, a relation between triangles called tetrahedron-connectedness, and so on, where these are defined as follows.

Two links are triangle-connected if there is a sequence of triangles, each overlapping the next in the sequence by two vertices. Similarly two triangles are tetrahedron-connected if there exists a sequence of tetrahedral skeletons, each overlapping the next in the sequence by three vertices. And in general 2 (n-1) simplices are n-connected if there exists a sequence of n-simplices, each overlapping the next in (n-1) vertices.

It is easy to see that just as link-connectedness is an equivalence relation between vertices, so triangle connectedness is an equivalence relation between links, tetrahedron-connectedness is an equivalence relation between triangles, and so on.

Each equivalence class of links, under triangle-connectedness defines a set of vertices. We define these sets as the subsystems of triangle-connected elements. Plainly, these subsystems are not necessarily disjoint. Again each equivalence class of triangles, under tetrahedron-connectedness, defines a set of vertices; we define these sets as the subsystems of tetrahedron-connected elements. These subsystems are again not disjoint; however, each of these subsystems is a subset of some triangle-connected subsystem. (This follows from the fact that two elements cannot be tetrahedron connected, unless they are triangle-connected).

The program EQCLA obtains all the triangle and tetrahedron connected subsystems of G.


ENTRY FROM MAIN
EQCLA
Preliminaries.
PXDD
Obtain all links in MATAX, and list them in MACRO.
TRY
Place first link in MACRO, into INMAT. If there are no links left in MACRO, decomposition is finished, so program ends.
TRIAN
Take end-elements of first link in INMAT, and place in X1 and X4. Then place in TEST all elements linked to both X1 and X4, and so forming triangles with X1 and X4.
UTEST
Put ORDER in IR4.
CLAX
Take next element from TEST, and place two links which this element makes with X1 and X4 respectively, in LOOK1 and LOOK4.
If either LOOK1 or LOOK4 is still in MACRO, take them out of MACRO and place them in INMAT.
NOT
If both LOOK1 and LOOK4 were in MACRO, place the triangle they define, in list of triangles in ATMS - (this procedure assures getting each triangle into ATMS just once).
Examine TEST
TEST still has elements in it
TEST is empty
When all elements from TEST have been examined, proceed to take next link from INMAT to construct new TEST, and continue.
When the last link from INMAT has been examined , without this adding any further links to INMAT, the contents of INMAT form an equivalence class of links.
PRINT
Take union of all elements which appear in any links in INMAT, and place this union in SET.
Then call PTSET to print out SET, under heading EQUIVALENCE CLASS OF LINKS.
TRTET
Now follows a section analogous to section TRY-TRIAN-UTEST-etc,which places triangles in INMAT instead of links, and constructs tetrahedra with next element from TEST, and moves remaining triangles in each of these tetrahedra, into INMAT, thus forming equivalence class of tetrahedron-connected triangles in INMAT.
Each such class is printed out under heading EQUIVALENCE CLASS OF TRIANGLES.
Each time ATMS (list of triangles) is empty, go back to TRY.
END OF PROGRAM
RETURN TO MAIN

```
*      FAP

                    00005          ENTRY   EQCLA

  TRANSFER VECTOR
00000  635131476360      TRIPT
00001  632563476360      TETPT
00002  476362256360      PTSET

  LINKAGE DIRECTOR
00003  000000000000
00004  255023432160


00005 -0634 00 4 00523   EQCLA SXD IR4,4
                         *THIS IS THE UNI GENERATOR
00006  0500 00 0 77461         CLA ORDER
00007  0622 00 0 00025         STD TXLL
00010  0774 00 1 00001         AXT 1,1
00011  0774 00 2 00001         AXT 1,2
00012 -0634 00 2 01547         SXD X2,2
00013  0774 00 4 00001         AXT 1,4
00014 -0500 00 4 77170   CALU  CAL UNIT,4
00015  0602 00 1 01134         SLW UNI,1
00016  0500 00 0 01547         CLA X2
00017  0622 00 1 01541         STD UNI2,1
00020  1 00001 1 00021         TXI *+1,1,1
00021  1 00001 4 00022         TXI *+1,4,1
00022 -3 00044 4 00014         TXL CALU,4,36
00023  1 00001 2 00024         TXI *+1,2,1
00024 -0634 00 2 01547         SXD X2,2
00025 -3 00000 1 00013   TXLL  TXL CALU-1,1,**
                         * NEXT SECTION LISTS ALL LINKS, ONE
                         *   WORD PER LINK, IN MACRO
00026 -0534 00 1 77461         LXD ORDER,1
00027 -0754 00 1 00000   PXDD  PXD 0,1
00030 -0734 00 4 00000         PDX 0,4
00031  0500 00 4 01541   CLAA  CLA UNI2,4
00032 -0734 00 2 00000         PDX 0,2
00033 -0500 60 1 76742         CAL* MATAX,1
00034 -0320 00 4 01134         ANA UNI,4
00035  0100 00 0 00045         TZE SKIP
00036 -0534 00 2 01542         LXD EDIR,2
00037  0634 00 4 01543         SXA EDGE,4
00040 -0634 00 1 01543         SXD EDGE,1
00041  0500 00 0 01543         CLA EDGE
00042  0601 00 2 47616         STO MACRO,2
00043  1 00001 2 00044         TXI *+1,2,1
00044 -0634 00 2 01542         SXD EDIR,2
00045  2 00001 4 00031   SKIP  TIX CLAA,4,1
00046  2 00001 1 00027         TIX PXDD,1,1
00047  0020 00 0 00050         TRA TRY
                         *END OF SECTION LISTING LINKS IN MACRO
                         *TRY TAKES NEXT NONZERO LINK FROM
                         *   MACRO TO START A NEW EQUIVALENCE
                         *   CLASS IN INMAT
00050 -0534 00 2 01542   TRY   LXD EDIR,2
00051  0600 00 2 66166         STZ INMAT,2
00052  2 00001 2 00051         TIX *-1,2,1
```

```
00053 -0534 00 2 01542          LXD EDIR,2
00054  0500 00 2 47616          CLA MACRO,2
00055 -0100 00 0 00060          TNZ *+3
00056  2 00001 2 00054          TIX *-2,2,1
00057  0020 00 0 00520          TRA OUT
00060  0600 00 2 47616          STZ MACRO,2
00061  0601 00 0 66165          STO INMAT-1
00062  0774 00 2 00002          AXT 2,2
00063 -0634 00 2 01545          SXD SCLIR,2
00064  0774 00 2 00001          AXT 1,2
00065 -0634 00 2 01544          SXD ECLIR,2
00066 -0634 00 2 00525          SXD TRIR,2
00067 -0534 00 2 01544    TRIAN LXD ECLIR,2
00070  0500 00 2 66166          CLA INMAT,2
00071  1 00001 2 00072          TXI *+1,2,1
00072 -0634 00 2 01544          SXD ECLIR,2
00073  0734 00 4 00000          PAX 0,4
00074 -0634 00 4 01550          SXD X4,4
00075 -0734 00 1 00000          PDX 0,1
00076 -0634 00 1 01546          SXD X1,1
00077 -0534 00 2 77460          LXD NWORD,2
00100 -0500 60 1 76742          CAL* MATAX,1
00101 -0320 60 4 76742          ANA* MATAX,4
00102  0602 00 2 01563          SLW TEST,2
00103  2 00001 2 00100          TIX *-3,2,1
00104  0020 00 0 00105          TRA UTEST
00105 -0534 00 4 77461    UTEST LXD ORDER,4
00106  0500 00 4 01541    CLAX  CLA UNI2,4
00107 -0734 00 2 00000          PDX 0,2
00110  0500 00 2 01563          CLA TEST,2
00111 -0320 00 4 01134          ANA UNI,4
00112  0100 00 0 00211          TZE NO
00113 -0754 00 4 00000          PXD 0,4
00114  0340 00 0 01546          CAS X1
00115  0020 00 0 00123          TRA LOK1
00116  0000 00 0 00000          HTR
00117  0500 00 0 01546          CLA X1
00120  0622 00 0 01564          STD LOOK1
00121  0634 00 4 01564          SXA LOOK1,4
00122  0020 00 0 00127          TRA LO4
00123 -0634 00 4 01564    LOK1  SXD LOOK1,4
00124  0500 00 0 01546          CLA X1
00125  0771 00 0 00022          ARS 18
00126  0621 00 0 01564          STA LOOK1
00127 -0754 00 4 00000    LO4   PXD 0,4
00130  0340 00 0 01550          CAS X4
00131  0020 00 0 00137          TRA LOK4
00132  0000 00 0 00000          HTR
00133  0500 00 0 01550          CLA X4
00134  0622 00 0 01565          STD LOOK4
00135  0634 00 4 01565          SXA LOOK4,4
00136  0020 00 0 00143          TRA LOKUP
00137 -0634 00 4 01565    LOK4  SXD LOOK4,4
00140  0500 00 0 01550          CLA X4
00141  0771 00 0 00022          ARS 18
00142  0621 00 0 01565          STA LOOK4
00143 -0534 00 2 01542    LOKUP LXD EDIR,2
00144  0500 00 2 47616          CLA MACRO,2
```

```
00145  0340 00 0 01564         CAS LOOK1
00146  0020 00 0 00150         TRA *+2
00147  0020 00 0 00152         TRA *+3
00150  2 00001 2 00144         TIX *-4,2,1
00151  0020 00 0 00157         TRA REP
00152  0600 00 2 47616         STZ MACRO,2
00153 -0534 00 2 01545         LXD SCLIR,2
00154  0601 00 2 66166         STO INMAT,2
00155  1 00001 2 00156         TXI *+1,2,1
00156 -0634 00 2 01545         SXD SCLIR,2
00157 -0534 00 2 01542   REP   LXD EDIR,2
00160  0500 00 2 47616         CLA MACRO,2
00161  0340 00 0 01565         CAS LOOK4
00162  0020 00 0 00164         TRA *+2
00163  0020 00 0 00166         TRA *+3
00164  2 00001 2 00160         TIX *-4,2,1
00165  0020 00 0 00173         TRA NOT
00166  0600 00 2 47616         STZ MACRO,2
00167 -0534 00 2 01545         LXD SCLIR,2
00170  0601 00 2 66166         STO INMAT,2
00171  1 00001 2 00172         TXI *+1,2,1
00172 -0634 00 2 01545         SXD SCLIR,2
                        * NEXT SECTION PUTS TRIANGLES, IF NEW,
                        *    INTO ATMS ( VERTICES OF TRIANGLES
                        *    APPEAR IN DESCENDING ORDER FROM LEFT
00173 -0754 00 4 00000   NOT   PXD 0,4
00174 -0625 00 0 00453         STL ST
00175  0020 00 0 00410         TRA X14
00176 -0534 00 2 00525         LXD TRIR,2
00177  0500 00 0 00524         CLA TRIWD
00200  0340 00 2 53536   CAST  CAS ATMS,2
00201  0020 00 0 00203         TRA *+2
00202  0020 00 0 00211         TRA NO
00203  0020 00 0 00204         TRA *+1
00204  2 00001 2 00200         TIX CAST,2,1
00205 -0534 00 2 00525         LXD TRIR,2
00206  0601 00 2 53536         STO ATMS,2
00207  1 00001 2 00210         TXI *+1,2,1
00210 -0634 00 2 00525         SXD TRIR,2
                        *END OF SECTION FOR STORING NEW TRIANGLES
00211  2 00001 4 00106   NO    TIX CLAX,4,1
00212  0020 00 0 00213         TRA NEWEC
00213  0500 00 0 01544   NEWEC CLA ECLIR
00214  0340 00 0 01545         CAS SCLIR
00215  0020 00 0 00220         TRA *+3
00216  0020 00 0 00221         TRA COMMT
00217  0020 00 0 00220         TRA *+1
00220  0020 00 0 00067         TRA TRIAN
00221  0074 00 4 00000   COMMT CALL TRIPT
00222  1 00000 0 00224
00223  0 00335 0 00003
00224  0020 00 0 00454         TRA PRINT-3
                        * TRTET TAKES NEXT NONZERO TRIANGLE
                        *    FROM ATMS TO START A NEW EQUIVALENCE
                        *    CLASS OF TRIANGLES
00225 -0534 00 2 00525   TRTET LXD TRIR,2
00226  0600 00 2 66166         STZ INMAT,2
00227  2 00001 2 00226         TIX *-1,2,1
```

```
00230 -0534 00 2 00525          LXD TRIR,2
00231  0500 00 2 53536          CLA ATMS,2
00232 -0100 00 0 00235          TNZ *+3
00233  2 00001 2 00231          TIX *-2,2,1
                     * TRIANGLES EXHAUSTED, SO GO BACK TO
                     *   NEXT EQUIVALENCE CLASS OF LINKS
00234  0020 00 0 00050          TRA TRY
00235  0600 00 2 53536          STZ ATMS,2
00236  0601 00 0 66165          STO INMAT-1
00237  0774 00 2 00002          AXT 2,2
00240 -0634 00 2 01545          SXD SCLIR,2
00241  0774 00 2 00001          AXT 1,2
00242 -0634 00 2 01544          SXD ECLIR,2
00243 -0534 00 2 01544    TETRA LXD ECLIR,2
00244  0500 00 2 66166          CLA INMAT,2
00245  1 00001 2 00246          TXI *+1,2,1
00246 -0634 00 2 01544          SXD ECLIR,2
00247  0601 00 0 00526          STO HOLD
00250 -0320 00 0 01570          ANA =0000777000777
00251  0734 00 4 00000          PAX 0,4
00252 -0734 00 1 00000          PDX 0,1
00253 -0534 00 2 77460          LXD NWORD,2
00254 -0500 60 1 76742          CAL* MATAX,1
00255 -0320 60 4 76742          ANA* MATAX,4
00256  0602 00 2 01563          SLW TEST,2
00257  2 00001 2 00254          TIX *-3,2,1
00260  0500 00 0 00526          CLA HOLD
00261  0771 00 0 00011          ARS 9
00262 -0320 00 0 01570          ANA =0000777000777
00263  0734 00 4 00000          PAX 0,4
00264 -0534 00 2 77460          LXD NWORD,2
00265 -0500 00 2 01563          CAL TEST,2
00266 -0320 60 4 76742          ANA* MATAX,4
00267  0602 00 2 01563          SLW TEST,2
00270  2 00001 2 00265          TIX *-3,2,1
00271  0020 00 0 00272          TRA UT
00272 -0534 00 4 77461    UT    LXD ORDER,4
00273  0500 00 4 01541          CLA UNI2,4
00274 -0734 00 2 00000          PDX 0,2
00275  0500 00 2 01563          CLA TEST,2
00276 -0320 00 4 01134          ANA UNI,4
00277  0100 00 0 00375          TZE NOL
00300  0500 00 0 00526          CLA HOLD
00301 -0320 00 0 01571          ANA =0000777777000
00302  0622 00 0 01546          STD X1
00303  0767 00 0 00011          ALS 9
00304  0622 00 0 01550          STD X4
00305 -0754 00 4 00000          PXD 0,4
00306 -0625 00 0 00453          STL ST
00307  0020 00 0 00410          TRA X14
00310 -0534 00 2 00525          LXD TRIR,2
00311  0500 00 2 53536          CLA ATMS,2
00312  0340 00 0 00524          CAS TRIWD
00313  0020 00 0 00315          TRA *+2
00314  0020 00 0 00317          TRA *+3
00315  2 00001 2 00311          TIX *-4,2,1
00316  0020 00 0 00324          TRA REP1
00317  0600 00 2 53536          STZ ATMS,2
```

```
00320 -0534 00 2 01545          LXD SCLIR,2
00321  0601 00 2 66166          STO INMAT,2
00322  1 00001 2 00323          TXI *+1,2,1
00323 -0634 00 2 01545          SXD SCLIR,2
00324  0500 00 0 00526    REP1   CLA HOLD
00325 -0320 00 0 01570          ANA =O000777000777
00326  0622 00 0 01546          STD X1
00327  0767 00 0 00022          ALS 18
00330  0622 00 0 01550          STD X4
00331 -0754 00 4 00000          PXD 0,4
00332 -0625 00 0 00453          STL ST
00333  0020 00 0 00410          TRA X14
00334 -0534 00 2 00525          LXD TRIR,2
00335  0500 00 2 53536          CLA ATMS,2
00336  0340 00 0 00524          CAS TRIWD
00337  0020 00 0 00341          TRA *+2
00340  0020 00 0 00343          TRA *+3
00341  2 00001 2 00335          TIX *-4,2,1
00342  0020 00 0 00350          TRA REP2
00343  0600 00 2 53536          STZ ATMS,2
00344 -0534 00 2 01545          LXD SCLIR,2
00345  0601 00 2 66166          STO INMAT,2
00346  1 00001 2 00347          TXI *+1,2,1
00347 -0634 00 2 01545          SXD SCLIR,2
00350  0500 00 0 00526    REP2   CLA HOLD
00351 -0320 00 0 01567          ANA =O000000777777
00352  0767 00 0 00011          ALS 9
00353  0622 00 0 01546          STD X1
00354  0767 00 0 00011          ALS 9
00355  0622 00 0 01550          STD X4
00356 -0754 00 4 00000          PXD 0,4
00357 -0625 00 0 00453          STL ST
00360  0020 00 0 00410          TRA X14
00361 -0534 00 2 00525          LXD TRIR,2
00362  0500 00 2 53536          CLA ATMS,2
00363  0340 00 0 00524          CAS TRIWD
00364  0020 00 0 00366          TRA *+2
00365  0020 00 0 00370          TRA *+3
00366  2 00001 2 00362          TIX *-4,2,1
00367  0020 00 0 00375          TRA NOL
00370  0600 00 2 53536          STZ ATMS,2
00371 -0534 00 2 01545          LXD SCLIR,2
00372  0601 00 2 66166          STO INMAT,2
00373  1 00001 2 00374          TXI *+1,2,1
00374 -0634 00 2 01545          SXD SCLIR,2
00375  2 00001 4 00273    NOL    TIX UT+1,4,1
00376  0500 00 0 01544          CLA ECLIR
00377  0340 00 0 01545          CAS SCLIR
00400  0020 00 0 00403          TRA *+3
00401  0020 00 0 00404          TRA COMT
00402  0020 00 0 00403          TRA *+1
00403  0020 00 0 00243          TRA TETRA
00404  0074 00 4 00001    COMT   CALL TETPT
00405  1 00000 0 00407
00406  0 00624 0 00003
00407  0020 00 0 00454          TRA PRINT-3
                        * SUBROUTINE FOR GETTING VERTICES WITHIN
                        *    A WORD IN DESCENDING NUMERICAL
```

```
                                     *   ORDER
00410   0600 00 0 00524   X14     STZ  TRIWD
00411   0340 00 0 01546           CAS  X1
00412   0020 00 0 00420           TRA  AA
00413   0000 00 0 00000           HTR
00414   0340 00 0 01550           CAS  X4
00415   0020 00 0 00430           TRA  BB
00416   0000 00 0 00000           HTR
00417   0020 00 0 00440           TRA  CC
00420  -0602 00 0 00524   AA      ORS  TRIWD
00421   0500 00 0 01546           CLA  X1
00422   0771 00 0 00011           ARS  9
00423  -0602 00 0 00524           ORS  TRIWD
00424   0500 00 0 01550           CLA  X4
00425   0771 00 0 00022           ARS  18
00426  -0602 00 0 00524           ORS  TRIWD
00427   0020 00 0 00450           TRA  STP
00430   0771 00 0 00011   BB      ARS  9
00431  -0602 00 0 00524           ORS  TRIWD
00432   0500 00 0 01546           CLA  X1
00433  -0602 00 0 00524           ORS  TRIWD
00434   0500 00 0 01550           CLA  X4
00435   0771 00 0 00022           ARS  18
00436  -0602 00 0 00524           ORS  TRIWD
00437   0020 00 0 00450           TRA  STP
00440   0771 00 0 00022   CC      ARS  18
00441  -0602 00 0 00524           ORS  TRIWD
00442   0500 00 0 01546           CLA  X1
00443  -0602 00 0 00524           ORS  TRIWD
00444   0500 00 0 01550           CLA  X4
00445   0771 00 0 00011           ARS  9
00446  -0602 00 0 00524           ORS  TRIWD
00447   0020 00 0 00450           TRA  STP
00450   0500 00 0 00453   STP     CLA  ST
00451   0400 00 0 01566           ADD  =1
00452   0601 00 0 00453           STO  ST
00453   0020 00 0 00000   ST      TRA  **
                                 * THE PRINTING SECTION FOLLOWS,
                                 *    PRINTING ENTIRE EQUIVALENCE CLASS
00454  -0534 00 2 77460           LXD  NWORD,2
00455   0600 00 2 77416           STZ  SET,2
00456   2 00001 2 00455           TIX  *-1,2,1
00457  -0534 00 2 01544   PRINT   LXD  ECLIR,2
00460   0500 00 2 66166           CLA  INMAT,2
00461  -0320 00 0 01570           ANA  =0000777000777
00462   0734 00 4 00000           PAX  0,4
00463   0500 00 4 01541           CLA  UNI2,4
00464  -0734 00 2 00000           PDX  0,2
00465  -0500 00 4 01134           CAL  UNI,4
00466  -0602 00 2 77416           ORS  SET,2
00467  -0534 00 2 01544           LXD  ECLIR,2
00470   0500 00 2 66166           CLA  INMAT,2
00471   0771 00 0 00011           ARS  9
00472  -0320 00 0 01570           ANA  =0000777000777
00473   0734 00 4 00000           PAX  0,4
00474   0500 00 4 01541           CLA  UNI2,4
00475  -0734 00 2 00000           PDX  0,2
00476  -0500 00 4 01134           CAL  UNI,4
```

```
00477 -0602 00 2 77416          ORS SET,2
00500 -0534 00 2 01544          LXD ECLIR,2
00501  0500 00 2 66166          CLA INMAT,2
00502 -0734 00 4 00000          PDX 0,4
00503  0500 00 4 01541          CLA UNI2,4
00504 -0734 00 2 00000          PDX 0,2
00505 -0500 00 4 01134          CAL UNI,4
00506 -0602 00 2 77416          ORS SET,2
00507  0500 00 0 01544          CLA ECLIR
00510  0402 00 0 77444          SUB ONED
00511  0100 00 0 00514          TZE CALLP
00512  0601 00 0 01544          STO ECLIR
00513  0020 00 0 00457          TRA PRINT
00514  0074 00 4 00002    CALLP CALL PTSET
00515  1 00000 0 00517
00516  0 01002 0 00003
00517  0020 00 0 00225          TRA TRTET
00520 -0534 00 4 00523    OUT   LXD IR4,4
00521  0020 00 4 00001          TRA 1,4
00522  0 00000 0 00000          PZE
00523  0 00000 0 00000    IR4
00524  0 00000 0 00000    TRIWD
00525  0 00000 0 00000    TRIR
00526  0 00000 0 00000    HOLD
00527  0 00000 0 00000    NEWT
01134                           BES 260
01134  0 00000 0 00000    UNI
01541                           BES 260
01541  0 00000 0 00000    UNI2
01542  0 00000 0 00000    EDIR
01543  0 00000 0 00000    EDGE
01544  0 00000 0 00000    ECLIR
01545  0 00000 0 00000    SCLIR
01546  0 00000 0 00000    X1
01547  0 00000 0 00000    X2
01550  0 00000 0 00000    X4
01563                           BES 10
01563  0 00000 0 00000    TEST
01564  0 00000 0 00000    LOOK1
01565  0 00000 0 00000    LOOK4
                          *   COMMON BLOCK FROM HIDECS 2
                 77462          COMMON -1
                 77462    INDIC COMMON 1
                 77461    ORDER COMMON 1
                 77460    NWORD COMMON 1
                 77457    DAT   COMMON 1
                 77456    LGTH  COMMON 1
                 77455    LATIS COMMON 1
                 77454    NBITH COMMON 1
                 77453    NBITL COMMON 1
                 77452    NBIT1 COMMON 1
                 77451    NBIT  COMMON 1
                 77450    NSQ1  COMMON 1
                 77447    OPRMN COMMON 1
                 77446    ATOMO COMMON 1
                 77445    ATOM  COMMON 1
                 77444    ONED  COMMON 1
                 77443    D36   COMMON 1
```

```
77442     ATOOX COMMON 10
77430     ATOX  COMMON 10
77416     SET   COMMON 10
77404     RANDM COMMON 10
77372     DIFF  COMMON 10
77360     CONVT COMMON 40
77310     DATA  COMMON 40
77240     MATA  COMMON 40
77170     UNIT  COMMON 40
77120     COMUN COMMON 40
77050     EQLS  COMMON 20
77024     SECTS COMMON 50
76742     MATAX COMMON 260
76336     DROWS COMMON 2100
72252     MROWS COMMON 2100
66166     INMAT COMMON 5400
53536     ATMS  COMMON 2000
47616     MACRO COMMON 7000
                END
```

```
  LITERALS
01566  000000000001
01567  000000777777
01570  000777000777
01571  000777777000

01572 IS THE FIRST LOCATION NOT USED BY THIS PROGRAM
```

The decomposition of the sample graph follows:

```
EQUIVALENCE CLASS OF LINKS
  1  2  3  4  5  6
EQUIVALENCE CLASS OF TRIANGLES
  4  5  6
EQUIVALENCE CLASS OF TRIANGLES
  2  3  4
EQUIVALENCE CLASS OF TRIANGLES
  2  4  5
EQUIVALENCE CLASS OF TRIANGLES
  1  2  3
EQUIVALENCE CLASS OF TRIANGLES
  1  2  5
EQUIVALENCE CLASS OF LINKS
  6  7  8  9 10 11
EQUIVALENCE CLASS OF TRIANGLES
  6  7  8  9 10 11
EQUIVALENCE CLASS OF LINKS
  4 11
EQUIVALENCE CLASS OF LINKS
 11 12
EQUIVALENCE CLASS OF LINKS
 12 13 14 15 16 17
EQUIVALENCE CLASS OF TRIANGLES
 14 15 16 17
EQUIVALENCE CLASS OF TRIANGLES
 13 14 15
EQUIVALENCE CLASS OF TRIANGLES
 12 13 15
EQUIVALENCE CLASS OF LINKS
  4 16
EQUIVALENCE CLASS OF LINKS
  1 17
```

CONCLUSION

All the four subprograms described are under the control of the program MAIN, which follows. There also follow listings of the two subsidiary subprograms, TETPT, and PTSET, which these programs require. All the other programs called by MAIN, BLDUP, STABL, SIMPX, EQCLA, are described, in detail, in the HIDECS 2 report. They are: INPAR, GENER, INDAT, CNDAT, SYMET, PTMAT, COUNT-CNVRT, PTLVL. The machine specification is the same as for HIDECS 2; IBM 7090, 32K core storage, MIT-FMS system; each of the four programs can handle systems of up to 250 elements. Finally, note that it is not possible to run more than one of these programs on the same calling sequence, because there is only room in core for storing one at a time.

```
  THIS IS MAIN PROGRAM
*      FAP

  TRANSFER VECTOR
00000  336225636447      .SETUP
00001  626346442147      STOMAP
00002  314547215160      INPAR
00003  272545255160      GENER
00004  314524216360      INDAT
00005  234524216360      CNDAT
00006  627044256360      SYMET
00007  476344216360      PTMAT
00010  626321224360      STABL
00011  256731636060      EXIT


00012  0074 00 4 00000          TSX $.SETUP,4
00013  0074 00 4 00001          CALL STOMAP
00014  1 00000 0 00016
00015  0 00015 0 00000
00016  0074 00 4 00002          CALL INPAR
00017  1 00000 0 00021
00020  0 00020 0 00000
00021  0074 00 4 00003          CALL GENER
00022  1 00000 0 00024
00023  0 00025 0 00000
00024  0074 00 4 00004          CALL INDAT
00025  1 00000 0 00027
00026  0 00030 0 00000
00027  0074 00 4 00005          CALL CNDAT
00030  1 00000 0 00032
00031  0 00033 0 00000
00032  0074 00 4 00006          CALL SYMET
00033  1 00000 0 00035
00034  0 00040 0 00000
00035  0074 00 4 00007          CALL PTMAT
00036  1 00000 0 00040
00037  0 00043 0 00000
00040  0074 00 4 00010          CALL STABL
00041  1 00000 0 00043
00042  0 00050 0 00000
                          * THE PREVIOUS CARD MAY BE REPLACED
                          *   BY CALL BLDUP, CALL SIMPX, OR
                          *   CALL EQCLA, WHICHEVER IS REQUIRED
                          * IT IS NOT POSSIBLE TO CALL MORE THAN
                          *   ONE OF THEM IN THE SAME CALLING
                          *   SEQUENCE, SINCE THERE IS NOT
                          *   ENOUGH ROOM IN CORE TO STORE THE
                          *   PROGRAMS THEMSELVES MORE THAN
                          *   ONE AT A TIME
00043  0074 00 4 00011          CALL EXIT
00044  1 00000 0 00046
00045  0 00053 0 00000
                          *   COMMON BLOCK FROM HIDECS 2
                 77462          COMMON -1
                 77462     INDIC COMMON 1
                 77461     ORDER COMMON 1
                 77460     NWORD COMMON 1
                 77457     DAT   COMMON 1
```

```
THIS IS MAIN PROGRAM

                    77456    LGTH  COMMON 1
                    77455    LATIS COMMON 1
                    77454    NBITH COMMON 1
                    77453    NBITL COMMON 1
                    77452    NBIT1 COMMON 1
                    77451    NBIT  COMMON 1
                    77450    NSQ1  COMMON 1
                    77447    OPRMN COMMON 1
                    77446    ATOMO COMMON 1
                    77445    ATOM  COMMON 1
                    77444    ONED  COMMON 1
                    77443    D36   COMMON 1
                    77442    ATOOX COMMON 10
                    77430    ATOX  COMMON 10
                    77416    SET   COMMON 10
                    77404    RANDM COMMON 10
                    77372    DIFF  COMMON 10
                    77360    CONVT COMMON 40
                    77310    DATA  COMMON 40
                    77240    MATA  COMMON 40
                    77170    UNIT  COMMON 40
                    77120    COMUN COMMON 40
                    77050    EQLS  COMMON 20
                    77024    SECTS COMMON 50
                    76742    MATAX COMMON 260
                    76336    DROWS COMMON 2100
                    72252    MROWS COMMON 2100
                    66166    INMAT COMMON 5400
                    53536    ATMS  COMMON 2000
                    47616    MACRO COMMON 7000
                                   END

00046 IS THE FIRST LOCATION NOT USED BY THIS PROGRAM


   NO ERROR IN ABOVE ASSEMBLY
* DATE AND TIME NOW      32/67 5498.1
```

```
*      FAP

                      00011            ENTRY    TETPT
                      00006            ENTRY    TRIPT

                               *   COMMENTS IN THE PROGRAM EQCLA

  TRANSFER VECTOR
00000   746263303460      (STH)
00001   742631433460      (FIL)

  LINKAGE DIRECTOR
00002   000000000000
00003   632563476360

00004   0 00000 0 00000          PZE
00005   0 00000 0 00013          PZE THREE
00006   0500 00 0 00005   TRIPT CLA *-1
00007   0020 00 0 00027         TRA START
00010   0 00000 0 00021         PZE FOUR
00011   0500 00 0 00010   TETPT CLA *-1
00012   0020 00 0 00027         TRA START
00013   602550643165      THREE BCI 6, EQUIVALENCE CLASS OF LINKS
00014   214325452325
00015   602343216262
00016   604626604331
00017   454262606060
00020   606060606060
00021   602550643165      FOUR  BCI 6, EQUIVALENCE CLASS OF TRIANGLES
00022   214325452325
00023   602343216262
00024   604626606351
00025   312145274325
00026   626060606060
00027   0621 00 0 00037   START STA LST1
00030  -0634 00 4 00051         SXD IR4,4
00031  -0500 00 0 00046         CAL TAPE2
00032   0074 00 4 00000         TSX $(STH),4
00033   0 00000 0 00047         PZE LELFT
00034  -0500 00 0 00050         CAL NUM
00035   0622 00 0 00042         STD TXH1
00036   0774 00 1 00000         AXT 0,1
00037   0560 00 1 00000   LST1  LDQ **,1
00040  -1 00000 0 00000         STR
00041   1 77777 1 00042         TXI *+1,1,-1
00042   3 00000 1 00037   TXH1  TXH LST1,1,**
00043   0074 00 4 00001         TSX $(FIL),4
00044  -0534 00 4 00051         LXD IR4,4
00045   0020 00 4 00001         TRA 1,4
00046   0 00002 0 00000   TAPE2 PZE 0,0,2
00047   740621063460      LELFT BCI 1,(6A6)
00050   0 77772 0 00000   NUM   PZE 0,0,-6
00051   0 00000 0 00000   IR4
                          *   COMMON BLOCK FROM HIDECS 2
                  77462         COMMON -1
                  77462   INDIC COMMON 1
                  77461   ORDER COMMON 1
                  77460   NWORD COMMON 1
```

```
77457    DAT    COMMON 1
77456    LGTH   COMMON 1
77455    LATIS  COMMON 1
77454    NBITH  COMMON 1
77453    NBITL  COMMON 1
77452    NBIT1  COMMON 1
77451    NBIT   COMMON 1
77450    NSQ1   COMMON 1
77447    OPRMN  COMMON 1
77446    ATOMO  COMMON 1
77445    ATOM   COMMON 1
77444    ONED   COMMON 1
77443    D36    COMMON 1
77442    ATOOX  COMMON 10
77430    ATOX   COMMON 10
77416    SET    COMMON 10
77404    RANDM  COMMON 10
77372    DIFF   COMMON 10
77360    CONVT  COMMON 40
77310    DATA   COMMON 40
77240    MATA   COMMON 40
77170    UNIT   COMMON 40
77120    COMUN  COMMON 40
77050    EQLS   COMMON 20
77024    SECTS  COMMON 50
76742    MATAX  COMMON 260
76336    DROWS  COMMON 2100
72252    MROWS  COMMON 2100
66166    INMAT  COMMON 5400
53536    ATMS   COMMON 2000
47616    MACRO  COMMON 7000
                END
```

```
*     FAP

                    CC004          ENTRY PTSET

  TRANSFER VECTOR
00000  746263303460     (STH)
00001  742631433460     (FIL)

  LINKAGE DIRECTOR
00002  00000000CCCC
00003  476362256360

00004 -0634 00 4 CC710   PTSET SXD IR4,4
                        *              PRELIMINARY
00005  050C C0 0 77461         CLA ORDER
00006  0622 00 0 CC017         STD TXL1
00007  0622 CC 0 CCC43         STD NOK
00010  0500 00 0 77460         CLA NWORD
00011  0622 00 0 00034         STD TXL3
                        *              GENERATE TABLE OF NUMBERS
00012  0774 00 1 CCC01         AXT 1,1
00013  0500 00 0 77444         CLA ONED
00014  0622 00 1 CC707         STD TABLE,1
00015  0400 00 0 77444         ADD ONED
00016  1 00001 1 CCC17         TXI *+1,1,1
00017 -3 00000 1 0C014   TXL1  TXL *-3,1,**
                        *              MODIFY TABLE TO INDICATE  SET
00020  0774 00 2 CC001         AXT 1,2
00021  0774 00 4 CCC01         AXT 1,4
00022  0774 00 1 00001   AXT1  AXT 1,1
00023  0560 00 2 77416         LDQ SET,2
00024  0162 00 0 00026         TQP *+2
00025  0020 00 0 CCC27         TRA *+2
00026  0600 00 4 C0707   STORE STZ TABLE,4
00027 -0773 00 0 CCCC1         RQL 1
00030  1 00001 1 CC031         TXI *+1,1,1
00031  1 00001 4 CCC32         TXI *+1,4,1
00032 -3 00044 1 CC024         TXL STORE-2,1,36
00033  1 00001 2 0C034         TXI *+1,2,1
00034 -3 00000 2 00022   TXL3  TXL AXT1,2,**
00035  0774 00 4 CCC01         AXT 1,4
00036  0774 00 1 00001         AXT 1,1
00037  0500 00 1 00707   CLAT  CLA TABLE,1
00040  0100 00 0 00043         TZE NOK
00041  0622 00 4 00707         STD TABLE,4
00042  1 C0001 4 00043         TXI *+1,4,1
00043  3 00000 1 00045   NOK   TXH FORM,1,**
00044  1 00001 1 CC037         TXI CLAT,1,1
00045  2 00001 4 CC046   FORM  TIX *+1,4,1
00046 -0634 00 4 00057         SXD TXL2,4
00047  0020 00 0 00050         TRA OUT
                        *              PRINT OUT MODIFIED TABLE.
00050 -0500 00 0 CC064   OUT   CAL NN
00051  0074 00 4 CCC00         TSX $(STH),4
00052  0 00000 0 00066         PZE FMT
00053  0774 00 1 0C001         AXT 1,1
00054  0560 00 1 CC707   LST2  LDQ TABLE,1
```

```
00055 -1 00000 0 00000          STR
00056  1 00001 1 00057          TXI  *+1,1,1
00057 -3 00000 1 00054    TXL2  TXL  LST2,1,**
00060  0074 00 4 00001          TSX  $(FIL),4
00061  0020 00 0 00062          TRA  FINIS
00062 -0534 00 4 00710    FINIS LXD  IR4,4
00063  0020 00 4 00001          TRA  1,4
00064  0 00002 0 00000    NN    PZE  0,0,2
00065  063103346060             BCI  1,6I3)
00066  740130607303       FMT   BCI  1,(1H ,3
00707                           BES  400
00707  0 00000 0 00000    TABLE
00710  0 00000 0 00000    IR4
00711  0 00000 0 00000    TWO
                         *   COMMON BLOCK FROM HIDECS 2
                 77462          COMMON -1
                 77462    INDIC COMMON 1
                 77461    ORDER COMMON 1
                 77460    NWORD COMMON 1
                 77457    DAT   COMMON 1
                 77456    LGTH  COMMON 1
                 77455    LATIS COMMON 1
                 77454    NBITH COMMON 1
                 77453    NBITL COMMON 1
                 77452    NBIT1 COMMON 1
                 77451    NBIT  COMMON 1
                 77450    NSQ1  COMMON 1
                 77447    OPRMN COMMON 1
                 77446    ATOMO COMMON 1
                 77445    ATOM  COMMON 1
                 77444    ONED  COMMON 1
                 77443    D36   COMMON 1
                 77442    ATOOX COMMON 10
                 77430    ATOX  COMMON 10
                 77416    SET   COMMON 10
                 77404    RANDM COMMON 10
                 77372    DIFF  COMMON 10
                 77360    CONVT COMMON 40
                 77310    DATA  COMMON 40
                 77240    MATA  COMMON 40
                 77170    UNIT  COMMON 40
                 77120    COMUN COMMON 40
                 77050    EQLS  COMMON '20
                 77024    SECTS COMMON 50
                 76742    MATAX COMMON 260
                 76336    DROWS COMMON 2100
                 72252    MROWS COMMON 2100
                 66166    INMAT COMMON 5400
                 53536    ATMS  COMMON 2000
                 47616    MACRO COMMON 7000
                                END
```

Christopher Alexander, Notes on the Synthesis of Form. Harvard University Press 1963 (a) Appendix 2.

Christopher Alexander, "The most stable decomposition of a system into subsystems", submitted to Information and Control, 1963 (b).

Christopher Alexander and Marvin Manheim, HIDECS 2: a computer program for the hierarchical decomposition of a set with an associated graph. Civil Engineering Systems Laboratory publication 160, MIT 1962.

Frank Harary and Ian C. Ross, "A procedure for clique detection using the group matrix", Sociometry 20 (Sept 1957), 205-15.

R. Duncan Luce, "Connectivity and generalised cliques in sociometric group structure", Psychometrika 15 (June 1950), 169-90.

Return to Sue Rogers
(Alexander's office)
(Arch Dept)
or [illegible]

HIDECS 3:
FOUR COMPUTER PROGRAMS FOR THE HIERARCHICAL DECOMPOSITION OF SYSTEMS WHICH HAVE AN ASSOCIATED LINEAR GRAPH

Christopher Alexander,

R63-27

June, 1963

MIT

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Cambridge 39, Massachusetts