Subsymmetries[1]

CHRISTOPHER ALEXANDER, *DEPARTMENT OF ARCHITECTURE, UNIVERSITY OF CALIFORNIA, BERKELEY*[2]
SUSAN CAREY, *CENTER FOR COGNITIVE STUDIES, HARVARD UNIVERSITY*

This paper concerns the cognitive simplicity of 35 black and white patterns. The patterns are rank-ordered according to four experimental measures of simplicity. The correlation between these different rank orders is high, and it is therefore possible to construct a single "simplicity" order for the patterns. It is then shown that the simplicity of different patterns is almost perfectly accounted for by the relative numbers of subsymmetries in the different patterns.

This paper describes the relative simplicity of 35 black and white patterns. Although each pattern is made of three black squares and four white squares, the patterns are different, and not all equally easy to perceive, conceive, remember, distinguish or describe.

We present four experiments. The first experiment determines which of the patterns can be *found* most quickly from a collection of patterns. The second experiment determines which of the patterns *seem* simplest to someone looking at them. The third experiment determines which of the patterns people find easiest to *remember,* and also which of the patterns are most likely to be *confused* with others. The fourth experiment determines which of the patterns are easiest to describe *in words.* Each of these experiments, then, defines an empirical measure of simplicity; and the third experiment defines two such measures. Each measure generates a rank order of simplicity on the 35 patterns. The experiments therefore generate five such rank orders.

There are two reasons for describing these rank orders. First, it turns out that the five rank orders are almost identical. One may, therefore, talk about the simplicity order of the patterns, and treat this rank order as a basic empirical fact about the cognition of these patterns. Second, this underlying simplicity order raises the question: "What stimulus properties are responsible for differences in cognitive simplicity?" The central result of this paper is this: *The cognitive simplicity of the 35 patterns is almost perfectly accounted for by the relative numbers of subsymmetries in the different patterns.*

EXPERIMENTAL PROCEDURE

Materials

The stimulus materials used in these experiments are the same as those described in Alexander and Huggins (1964). There are 35 patterns, each printed on a strip of paper 2-5/8 in. long and 3/8 in. high. Each pattern is, in effect, a horizontal linear arrangement of three black and four white squares, but adjacacent squares of the same color are not separated. Along the bottom edge of the pattern there is a thin grey line, which orients the pattern, and keeps mirror-image patterns distinct. There are just 35 different possible arrangements of this kind (7!/3!.4!), and each of these possible arrangements appears just once among the 35 stimuli. To keep figure-group relationships constant during the experiments, the patterns were always seen against the same achromatic grey, chosen so that neither the black nor the white seemed to stand out more strongly than the other. The patterns are illustrated in Fig. 1. They are shown in the order of simplicity determined by the five experiments.

Two random-like arrays of all 35 patterns, called R2 and R3, were used. Each of these random-like arrays is a rectangular block of patterns, five by seven, the patterns arranged in such a way that there is no discernible rule or regularity governing their positions, with adjacent patterns far enough apart to prevent visual interference. Since the position of a pattern in an array can make it easier to find, or easier to remember, it is important to control for this effect. R2 and R3 were, therefore, constructed to be complementary, so that for any pattern, the position effects are reversed in the two arrays.[3]

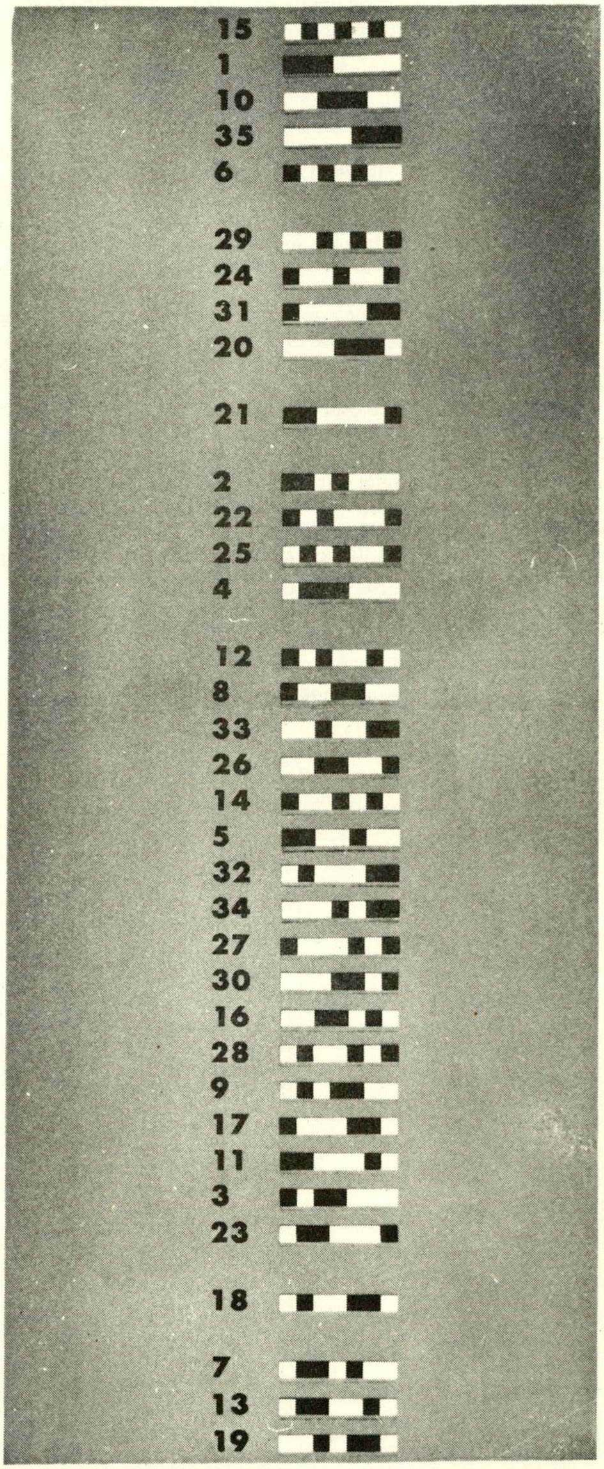

Fig. 1. Stimuli. They are shown in the overall empirical simplicity order, based on the sum of the ranks given by the individual experiments. The gaps reflect big jumps in the sum of the ranks. The numbers are merely labels, for identification only.

Each random-like array and each individual pattern was photographed on 35 mm slides. These slides could be back projected, at life size, preserving the proper black-white-grey relationships, onto a vertical milk-glass screen in front of S.

Subjects

The Ss were all undergraduate girls from Radcliffe College. None of the Ss in Experiments 1 (N=22), 2 (N=12), 3 (N=28), had ever seen the stimulus materials before. The Ss in Experiment 4 (N=5) had all been Ss in one of the earlier experiments.

EXPERIMENT 1: SEARCH (N = 22)

This experiment was originally developed for a different purpose (Alexander & Huggins, 1964).

The S sat at a table. In front of her, on the table, was one of the random-like arrays. Beyond the array, about 2 ft from the S, was the vertical milk-glass screen, onto which the E could project images of single patterns. In the slide projector was a tray containing 35 slides, one of each pattern, in a pre-set random order. One pattern at a time was projected on the screen and held there until the S pointed to the same pattern on the random-like array. S was told she would be timed: E stopped the clock when she pointed to the correct pattern.

Each S ran through the procedure with three arrays. Eleven Ss were shown R3, R2, R3 and another 11 were shown R2, R3, R2. In all cases the first run through was a warm up, designed to eliminate from the results the very sharp drop in search time which occurred during the first few patterns. By the time S reached her second array, the average search time was stable. The search time for a pattern defines a measure of simplicity. On the basis of this measure, each complete run through on a random-like array yields one rank order of simplicity for the 35 patterns. The experiment yielded a total of 22 orders for R2 and 22 for R3.

EXPERIMENT 2: SUBJECTIVE (N = 12)

This was a very informal experiment. In one room a random-like array was set up on the floor. The fact that it was on the floor forced the S to survey the whole array at once, preventing her from peering only at a small section of it. In another room was a blank grey board, like the one on which the random array was set up, but with the patterns scattered aimlessly around the edge of the board. The S was instructed to choose one pattern at a time, memorize it and its position, and then go to the other room, find the same pattern and put it in the appropriate position on the board. She was asked to do this in the order of the simplicity of the patterns. She was requested to ignore the fact that the positions of some patterns would obviously be easier to remember than others. Each S did the task twice, once for R2, once for R3. Six did R2 first, six did R3 first. We define the order in which the S chose the patterns as the simplicity order for this task. This experiment yielded 12 orders for R2 and 12 orders for R3.

EXPERIMENT 3: MEMORIZATION AND CONFUSION (N = 28)

Each S sat at a table. In front of her was the milk-glass screen. E handed S a card with four patterns on it. S was told that she had 30 sec to memorize the patterns; they were covered up after the 30 sec elapsed. Ten sec after that, she would be asked to choose the same four patterns from a large number of similar ones (in fact, a random-like array) that would be projected onto the screen. She was encouraged in two ways to be sure to remember the patterns which were easiest. She was told that E was only interested in exact identifications; there would be no credit for near misses. Therefore, she was asked to point first to the pattern she was surest of and was warned not to miss easy ones just because she thought she "had" them. To insure against this possibility, she was instructed to close her eyes for a few seconds halfway through the 30 sec memorization time to check herself on the patterns she "had."

There were 35 sets of four patterns (quartets), so that each pattern appeared in four quartets, once in each position. The quartets were presented in such an order that no two succeeding quartets contained the same pattern. Fourteen Ss worked with R2 and 14 with R3. A S, in the course of an hour, could complete about 18 quartets. It was, therefore, not possible to test

Table 1
Summary of Correlation Coefficients and Levels of Significance of Correlations between Rank Orders Produced in each Separate Experiment

		Concordances between rank order produced by individual subjects		Correlations between combined rank orders for R2 and combined rank orders for R3	
Experiment		W (Kendall's coefficient of concordance)*	level of significance	Spearman's rho	level of significance
1 Search	R2 (n = 22 rankings)	.263	.000001	.521	.003
	R3 (n = 22 rankings)	.327	.000001		
2 Subjective	R2 (n = 12 rankings)	.474	.00001	.800	.00001
	R3 (n = 12 rankings)	.546	.0001		
3a Memorization	n = 2 rank orders, each a composite of 14 subjects	**		.678	.0001
3b Confusion	n = 2 rank orders, each a composite of 14 subjects	**		.749	.00001
4 Verbal	n = 5 rank orders	*** .438	.001		

**(Kendall, 1948)*

***In experiment 3 it was impossible to test subject to subject agreement because no single subject ranked all 35 patterns (see text).*

****Corrections for tied rankings have been made (Kendall, 1948)* subject-to-subject agreement by comparing individual rank orders. Instead we obtained measures by summing over 14 Ss.

This experiment produced two measures of simplicity. The memorization measure is defined by the number of times a pattern was identified correctly, summed for 14 Ss. The confusion measure is defined by the number of times a pattern was chosen wrongly (i.e., had in fact *not* been one of the four patterns presented to her), again summed for 14 Ss. The memorization measure yields two rank orders, one for the 14 R2 Ss, and one for the 14 R3 Ss. The confusion measure also yields two rank orders, one for the 14 R2 Ss, and one for the 14 R3 Ss.

EXPERIMENT 4: VERBAL DESCRIPTION (N = 5)

Two Ss sat on opposite sides of an opaque screen, so that they could hear one another, but not see. One (A) had R2 in front of her; the other (B) had R3 in front of her. A was given a card with five patterns on it. She had 30 sec to describe these five patterns to B, during which time B listened with closed eyes. B then tried to point out on her own random- like array the five patterns that A had described, and A was told which ones B got right. Then A and B changed roles, and B described a quintet to A. Each S described a total of seven quintets to her partner. Every seven quintets were chosen to contain each pattern once, and balanced for position effects.

There were five Ss in this experiment, each paired with each of the four others, thus making a total of 10 experimental runs. Each S described every pattern just once to each of her four partners and thus had a total of four opportunities to communicate the pattern correctly. The simplicity measure is defined by the number of correct communications. Every pattern, therefore, gets a score of between 0 and 4 for each S. This measure yields a simplicity order for each of the five Ss.

RESULTS

The experimental data indicate the presence of a single underlying order of cognitive simplicity for the 35 patterns. The argument has three steps.

First, we test subject-to-subject agreement in any one experiment. If subject-to-subject agreement is high, we may then combine individual Ss ranks, to obtain an aggregate rank order for a given experimental procedure.

Second, in those experiments where arrays of patterns are used, we show that the array used, and hence the position of a pattern in the array, does not substantially affect the results.

These two steps are presented in Table 1. The table shows that subject-to-subject agreement is high; and that the array used has very little effect. It is, therefore, reasonable to construct a single composite rank order for each experiment. These five composite rank orders are shown in Table 2.

The third step, showing the correlations between the five rank orders, is shown in Table 3. Kendall's coefficient of concordance for the five rank orders, is .647, which is significant beyond the .00001 level. The fact that the individual pairwise correlations shown in Table 3 are all approximately equal, makes the overall

Table 2
Composite Ranks from each of the Experiments Shown with the Overall Rank Order

Pattern's Identification Number	Rank On Experiment 1 Search	Rank On Experiment 2 Subjective	Rank On Experiment 3 Memorization	Rank On Experiment 3 Confusion	Rank On Experiment 4 Verbal	OVERALL RANK
15	2	3	1	2	2½	1
1	3	1½	3½	4	4	2
10	7	5	2	1	1	3
35	1	1½	3½	5½	9	4
6	6	9	7	3	2½	5
29	14	9	5	7½	13½	6
24	22	9	6	5½	9	7
31	10	5	16	7½	13½	8
20	4	5	8	11	28½	9
21	11	9	18	12	16½	10
2	5	25	14	23	9	11
22	8	20	26½	16	9	12
25	16	13	11	21½	20½	13
4	26	9	9	9	28½	14
12	17½	12	20½	25½	16½	15
8	25	22	23	13½	9	16
33	13	23	20½	16	20½	17
26	27	15	24½	21½	9	18
14	20	14	35	24	9	19
5	17½	27	26½	13½	20½	20
32	15	18	20½	28	25	21
34	19	29	28½	10	23	22
27	34	28	12½	19½	16½	23
30	28	17	24½	16	25	24
16	23	33	15	35	5	25
28	31	32	10	18	20½	26
9	35	19	12½	32	16½	27
17	12	24	20½	29½	32	28
11	9	26	32	27	28½	29
3	21	16	34	19½	32	30
23	24	21	30½	25½	25	31
18	29	31	17	29½	28½	32
7	30	34	28½	34	34½	33
13	33	30	33	32	32	34
19	32	35	30½	32	34½	35

Table 3
Spearman's Rhos for the Pairwise Correlations of the Five Empirical Rank Orders

	S	Su	M	C	V
Search		.63	.45	.58	.45
Subjective			.64	.74	.50
Memorization				.62	.54
Confusion					.55
Verbal					

correlation even more significant. Four of the pairwise correlations are independent (for instance, the four correlations between C and S, Su, M, V). Since each is significant beyond the .003 level, the chance of getting five rank orders with these correlations is less than .0000001.

We may, therefore, assume that these five rank orders reflect a single underlying rank order: and we obtain a best estimate of this underlying order by ordering the patterns according to the sums of the ranks on these five orders. This overall order is shown in Table 3, and illustrated in Fig. 1. We shall call this overall order *the* simplicity order of the patterns.[4]

DISCUSSION

The question now arises: What stimulus properties make a pattern more or less simple?

Study Fig. 1, and try to find a stimulus property which explains the rank order of the patterns. One property suggests itself at once: *symmetry*. The simplest pattern of all is a symmetrical one, and all the symmetrical patterns are high on the simplicity order. But there are counterexamples too: the second simplest pattern is as asymmetrical as it can be. Besides, the concept of symmetry applies to only three of the 35 patterns. It does not tell us why the other 32 patterns should have any systematic differences in simplicity among them. Another obvious property is the number of blocks in the pattern. The second pattern has only two blocks in it, one black and one white, and all those patterns with few blocks are high on the simplicity order. But again there are startling counterexamples: the simplest pattern of all has as many blocks as it can have—seven. And if we rank order the patterns according to the number of blocks they contain, this rank order has a correlation of only .198 with the observed simplicity order. We might expect some simple combination of *symmetry* and *number of blocks* to account for simplicity: but according to this hypothesis the symmetrical pattern consisting of one black block, flanked on each side by one white block, would be the simplest. It is not.

It seems plain that neither of these concepts, nor any simple combination of them explains the rank order. Yet these two concepts are the only well defined concepts so far proposed by psychological theory that could explain the simplicity of these patterns. The Gestalt theorists proposed "good Gestalt" as an explanation of simplicity: but the only well defined Gestalt property which can be applied to these patterns is symmetry (Kohler, 1929; Koffka, 1935; Wulf, 1939; Mowatt, 1940).

Attneave has also proposed symmetry (which he treats as redundancy), as a measure of simplicity (1955). Garner and Clement (1963) have proposed the number of distinct patterns which are isomorphic under reflection and rotation, as a measure of simplicity. This is again symmetry. Attneave (1954; 1957) has proposed the number of changes encountered by a perceiver who scans the pattern, as a measure. This is one less than the number of blocks, and yields the same rank order. Miller (1956) has proposed the number of perceived "chunks" as the factor governing immediate memory (again the number of blocks).

Yet neither symmetry nor number of blocks explain the rank order: In the first two patterns they even seem contradictory. We, therefore, seek some deeper, more general structural property, which yields both symmetry and large blocks as special cases. More concretely: can we find some structural property which is shared by the symmetrical pattern of seven blocks and by the asymmetrical pattern of two blocks?

The structural property we seek is "the number of subsymmetries in the pattern." This property is shown against the empirical simplicity order in Table 4. The rank order it generates, has a correlation of .808 with the empirical simplicity order. This correlation is significant at the .00001 level. As far as we can tell, the number of subsymmetries in a pattern explains its perceived simplicity.

We define subsymmetries as follows. First, we define a segment. Each of the 35 patterns considered in this paper, consists of seven squares in a row, every square colored black or white. A segment of a pattern is any connected set of two or more squares within the pattern, regardless of color. Every pattern contains just 21 segments. (One segment 7 squares long, two segments 6 squares long, three segments 5 squares long, four segments 4 squares long, five segments 3 squares long, and six segments 2 squares long.) Next, we define a symmetry. A symmetry is an operation which maps a pattern onto itself, in such a way as to preserve distance between points and color (technically this is called a colored symmetry, Shubnikov, 1964). Since the patterns in this paper are essentially one-dimensional, we shall restrict attention to bilateral symmetries. Each whole pattern has just one such symmetry, or none.

It is usual to apply the concept of symmetry only to whole patterns; but we may also apply it to segments within a pattern. Each of the 21 segments within a pattern may itself be either symmetrical or asymmetrical, according to its coloring. Thus, in the case of the simplest pattern (ID No. 15) the 3, 5, and 7-square segments are all symmetrical, and the 2, 4 or 6-square segments are all asymmetrical. In the case of the second simplest pattern (ID No. 1) five of the 2-square segments, one 4-square segment, and three 3-square segments, are symmetrical and all the others, are asymmetrical.

Table 4

Pattern Identification Number	Empirical Rank	Number of Subsymmetries
15	1	9
1	2	9
10	3	7
35	4	9
6	5	7
29	6	7
24	7	7
31	8	8
20	9	7
21	10	8
2	11	6
22	12	6
25	13	6
4	14	7
12	15	6
8	16	6
33	17	6
26	18	6
14	19	6
5	20	6
32	21	6
34	22	6
27	23	6
30	24	6
16	25	5
28	26	6
9	27	5
17	28	6
11	29	6
3	30	6
23	31	6
18	32	5
7	33	5
13	34	5
19	35	5

We define a subsymmetry of a pattern, then, as a bilateral symmetry of a segment within that pattern. Since each segment has just one symmetry, or none, the number of subsymmetries in a pattern is the same as the number of symmetrical segments.

The fundamental result of this paper is: *Patterns with many subsymmetries are cognitively simple. Patterns with few subsymmetries are not cognitively simple.*[5] The number of subsymmetries in each pattern, and the rank order generated by the number of subsymmetries, is shown in Table 4, alongside the empirical simplicity order. The simplest patterns have nine subsymmetries in them. The least simple patterns have five subsymmetries in them. As noted before, the correlation between the two rank orders is .808, with a significance level better than .00001.

It seems highly likely that the set of subsymmetries of a pattern plays a major part in the way that the pattern is represented in the brain. Is a pattern perhaps even completely represented by the interlock of its subsymmetries? Are there neural nets in the visual system designed to compute the whole set of subsymmetries for a given input? These are open questions.

REFERENCES

ALEXANDER, C., & HUGGINS, A. W. F. On changing the way people see. *Percept. mot. Skills,* 1964, 19, 235-253.

ATTNEAVE, F. Physical determinants of the judged complexity of shapes. *J. exp. Psychol.,* 1957, 53, 221-227.

ATTNEAVE, F. Symmetry, information, and memory for patterns. *Amer. J. Psychol.,* 1955, 68, 209-22.

ATTNEAVE, F. Some informational aspects of visual perception. *Psychol. Rev.,* 1954, 61, 183-193.

GARNER, W. R., & CLEMENT, D. E. Goodness of pattern and pattern uncertainty. *J. verbal Learn. verbal Behav.,* 1963, 2, 446-452.

KENDALL, M. G. *Rank correlation methods.* London: Charles Griffin & Co., 1948.

KOFFKA, K. *Principles of Gestalt psychology.* London: Geo. Routledge & Son Ltd., 1935.

KÖHLER, W. *Gestalt psychology.* New York: H. Liveright, 1929.

MILLER, G. A. The magical number seven, plus or minus two: Some limits on our capacity for processing information. *Psychol. Rev.,* 1956, 63, 81-97.

MOWATT, M. H. Configurational properties considered "good" by naive subjects. *Amer. J. Psychol.,* 1940, 53, 46-69.

SHUBNIKOV, A. V., BELOV, N. V., et al. *Colored symmetry.* New York: Pergamon Press, 1964.

WULF, F. Tendencies in figural variation. In W. D. Ellis (Ed.), *A source book of Gestalt psychology.* New York: Harcourt, Brace. 1939. Pp. 136-148.

NOTES

1. Almost all the experiments described in this paper were run by Edward Schmookler, Harvard College. I am very grateful to him for his careful work, and for his comments on the exe
and for his comments on the experiments.

2. Formerly of the Center for Cognitive Studies, Harvard University, Cambridge, Massachusetts.

3. They are illustrated in Alexander and Huggins, 1964, p. 238.

4. It is worth pointing out, that the rank orders obtained from the subjective and confusion experiments have the highest correlation with the overall simplicity order, and that these experiments, of the four reported, are therefore the best estimators of *the* simplicity order.

5. It must be noted that all the patterns in our experiments have the same relative amount of black and white in them (i.e., three black squares and four white squares). To compare the simplicity of patterns which contain different relative amounts of black and white, it may or may not be necessary to modify the number of subsymmetries.

(Accepted for publication December 26, 1967.)