THE CITY IS A SEMI-LATTICE, BUT NOT A TREE

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I. TREES AND SEMI-LATTICES

The tree of my title is not a green tree with leaves. It is the name for a pattern of thought. The semi-lattice is the name for another, more complex, pattern of thought.

Both the tree and the semi-lattice are ways of thinking about how a large collection of many small systems go to make up a large and complex system. More generally, they are both names for structures of sets. A tree is a specially simple kind of semi-lattice; a semi-lattice, being the more general of the two, is not necessarily a tree.

In order to define such structures, let me first define the concept of a set. A set is a collection of elements which for some reason we think of as belonging together. Since, as designers, we are concerned with the physical living city, and its physical backbone, we most naturally restrict ourselves to considering sets which are collections of material elements, like people, bricks, molecules, houses, gardens, water pipes, the water molecules that run in them, etc.

When the elements of a set belong together because they cooperate, or work together somehow, we call the set of elements a system.

Take an example:

In Berkeley, at the corner of Hearst and Euclid, there is a drug store, and outside the drug store, a traffic light. In the entrance to the drug store there is a newsrack where the day's papers are displayed. When the light is red, people who are waiting to cross the street stand idly by the light; and since they have nothing to do, they look at the papers displayed on the newsrack, which they can see from where they stand. Some of them just read the headlines; others actually buy a paper while they wait. This effect makes the newsrack and the traffic light interdependent; the news-rack, the newspapers on it, and the money going from people's pockets to the

dime slot, the people who stop at the light and read papers, the traffic light, the electric impulses which make the lights change, and the sidewalk which the people stand on form a system--they work together.

From the designers point of view, the physically unchanging part of this system is of special interest. The newsrack, the traffic light, and the sidewalk between them, related as they are, from the fixed part of the system. It is the unchanging receptacle in which the changing parts of the system--people, newspapers, money, and electrical impulses--can work together. I define this fixed part as a unit of the city. It derives its coherence as a unit, both from the forces which hold its own elements together, and from the dynamic coherence of the larger living system which includes it as a fixed invariant part.

Other examples of systems in the city are: the set of particles which go to make up a building; the set of particles which go to make up a human body; the cars on the freeway, plus the people in them, plus the freeway they are driving on; two friends on the phone, plus the telephones they hold, plus the telephone line connecting them; Telegraph Hill with all its buildings, services and inhabitants; the chain of Rexall drug stores; the physical elements of San Francisco that fall under the administrative authority of City Hall; everything within the physical boundary of San Francisco, plus all the people who visit the city regularly and contribute to its development (like Bob Hope or the president of Arthur D. Little), plus all the major sources of economic welfare which supply the city with its wealth; the dog next door, plus my garbage can, plus the garbage out of my garbage can which he lives on; the San Francisco chapter of the John Birch Society.

Each one of these is a set of elements made coherent and cooperative by some sort of inner binding forces. And each one, just like the traffic

light—newsrack system, has a physically fixed part which we may regard as a unit of the city.

Of the many, many fixed concrete subsets of the city which are the receptacles for its system, and can therefore be thought of as significant physical units, we usually single out a few for special consideration. In fact, I claim that whatever picture of the city someone has is defined precisely by the subsets he sees as units.

Now, a collection of subsets which goes to make up such a picture is not merely an amorphous collection. Automatically, merely because relationships are established among the subsets, once the subsets are chosen, the collection has a definite structure. To understand this structure, let us think abstractly for a moment. Instead of talking about the real sets of millions of real particles which occur in the city, let us consider a simpler structure made of just half a dozen elements. Label these elements 1, 2, 3, 4, 5, 6. Not including the full set (1, 2, 3, 4, 5, 6), the empty set ( ), and the one element sets, (1), (2), (3), (4), (5), (6), there are $2^6 - 8 = 56$ different subsets we can pick from 6 elements. Suppose we now pick out certain of these 56 sets (just as we pick out certain sets and call them units when we form our picture of the city). Let us say, for the sake of example, that we pick the following subsets: (123), (34), (45), (234), (345), (12345), (3456).

What are the possible relationships among these sets? Some sets will be entirely part of larger sets, as (34) is part of (345) and (3456). Some of the sets will overlap, like (123) and (234). Some of the sets will be disjoint, that is, contain no elements in common, like (123) and (45).

We can see these relationships displayed in two ways. In figure A1 each set chosen to be a unit has a line drawn round it. In figure A2 the chosen sets are arranged in order of ascending magnitude, so that whenever

one set contains another (as 345 contains 34), there is a vertical path leading from one to the other. For the sake of clarity and visual economy, it is usual to draw lines only between sets which have no further set intermediate between them (thus the line between 34 and 345, and the line between 345 and 3456, make it unnecessary to draw a line between 34 and 3456).

As we see from these two representations, the choice of subsets alone endows the collection of subsets as a whole with an over-all structure. This is the structure which we are concerned with in this lecture. When the structure meets certain conditions it is called a semi-lattice. When it meets other, more restrictive conditions, it is called a tree.

Semi-lattice axiom: A collection of sets forms a semi-lattice if and only if two overlapping sets belong to the collection; then the set of elements common to both also belongs to the collection.

The structure illustrated in figure A is a semi-lattice. It satisfies the axiom since, for instance, 234 and 345 both belong to the collection, their common part, 34, also belongs to it. (As far as the city is concerned, this axiom states merely that wherever two units overlap, the area of overlap is itself a recognizable entity and hence a unit also. In the case of the drug store example, one unit consists of the newsrack, sidewalk, and traffic light. Another unit consists of the drug store itself, with its entry and the newsrack. The two units overlap in the newsrack. Clearly this area of overlap is itself a recognizable unit, and so satisfies the semi-lattice axiom.)

Tree axiom: A collection of sets forms a tree if and only if any two sets belong to the collection, either one is wholly contained in the other, or else they are wholly disjoint.

The structure illustrated in figure B is a tree. Since this axiom excludes the possibility of overlapping sets, there is no way in which the semi-lattice axiom can be violated, so that every tree is, trivially, a semi-lattice.

However, in this lecture we are not so much concerned with the fact that a tree happens to be a trivially simple semi-lattice, as with the difference between trees and those more general semi-lattices which are not trees because they do contain overlapping units. We are concerned with the difference between structures in which no overlap occurs, and those structures in which overlap does occur.

It is not merely the overlap which makes the distinction between the two important. Still more important, is the fact that the semi-lattice is potentially a much more complex and subtle structure than a tree. We may see just how much more complex a semi-lattice can be than a tree in the fact that a tree based on 20 elements can contain at most 19 further subsets of the 20 while a semi-lattice based on the same 20 elements can contain more than 1,000,000 different further subsets. This enormously greater variety is an index of the great structural complexity a semi-lattice can have when compared with the structural simplicity of a tree. It is this lack of structural complexity, characteristic of trees, which is crippling our conceptions of the city.

II. NATURAL AND ARTIFICIAL CITIES

In order to relate these abstract thoughts to the nature of the city, I must first make a simple distinction. I want to call those cities which have arisen more or less spontaneously over many, many years, natural cities. And I shall call those cities and parts of cities which have been deliberately created by designers and planners, artificial cities. Siena, Liverpool, Kyoto, Manhattan, are examples of natural cities. Levittown, Chandigher, and the British New Towns, are examples of artificial cities.

It is more and more widely recognized today, that there is some essential ingredient missing from artificial cities. When compared with ancient cities that have acquired the patina of life, our modern attempts to create cities artificially are, from a human point of view, entirely unsuccessful.

Architects themselves admit more and more freely that they really like living in old buildings more than new ones. The non-art-loving public at large, instead of being grateful to architects for what they do, regards the onset of modern buildings and modern cities everywhere as an inevitable, rather sad, piece of the larger fact that the world is going to the dogs.

It is much too easy to say that these opinions represent only people's unwillingness to forget the past, and their determination to be traditional. For myself, I trust this conservatism. Americans are usually willing to move with the times. Their growing reluctance to accept the modern city evidently expresses a longing for some real thing, something which for the moment escapes our grasp.

The prospect that we may be turning the world into a place peopled only by little glass and concrete boxes has alarmed many architects too. To combat the glass box future, many valiant protests and designs have been put forward, all hoping to re-create in modern form, the various characteristics of the natural city which seem to give it life. But so far these

protests and designs have only remade the old, they have not been able to create the new.

"Outrage," the Architectural Review's campaign against the way in which new construction and telegraph poles are wrecking the English town, based its remedies, essentially, on the idea that the spatial sequence of buildings and open spaces must be controlled, if scale is to be preserved—an idea that really derives from Camillo Sittes' book about ancient squares and piazzas. Another attempt tries to recapture the rich shapes of an old town, in protest against the monotony of Levittown, tries to recapture the richness of shape found in the houses of a natural old town. Llewellyn Davies' vissage at Rushbrooke in England is an example—each cottage is slightly different from its neighbor, the roofs jut in and out at picturesque angles: the shapes are "interesting" and cute. Another recent idea is to get high density back into the city. The idea behind these schemes seems to be that if the whole metropolis could only be like Grand Central Station, with lots and lots of layers and tunnels all over the place, and enough people milling around in them, maybe it will be human again. The artificial urbanity of Victor Gruen's schemes, Penn Center, Tanges MIT project, and the LCC's scheme for Hook New Town, all betray this thought at work. Another very brilliant critic of the deadness which is everywhere, is Jane Jacobs. Her criticisms are excellent. But when you read her concrete proposals for what we should do instead, you get the idea that she wants the great modern city to be a sort of mixture between Greenwich village and some Italian hill town, with lots of short blocks and lots of people sitting in the street.

The problem these designers have tried to face is real. It is vital that we discover the property of old towns which gave them life, and get it back into our own artificial cities. But we cannot do this merely by

remaking English villages, Italian piazzas, and Grand Central Stations. Too many designers today seem to be yearning for the physical and plastic characteristics of the past, instead of searching for the abstract ordering principle which the towns of the past happened to have, and which our modern conceptions of the city have not yet found. These designers fail to infuse new life into the city, because they imitate merely the appearance of the old, its concrete substance: they fail to unearth its inner nature.

What is the inner nature, the ordering principle, which distinguishes the artificial city from the natural city?

You will have guessed from my title, what I believe this ordering principle to be. I believe that a natural city has the organization of a semi-lattice: but that when we organize a city artificially, we organize it as a tree.

To persuade you to agree with me, I must first show by example that it is true; and I must then explain why it is happening—why it is that such a violently different structure comes about when a city is artificially conceived.

I shall first try to establish clearly what I mean by saying that artificial cities have a tree structure, so that we are all talking about the same thing, and I shall try to prove that it is so. Then in the section following I shall try to show how the living city cannot be properly contained in a receptacle which is a tree—that indeed its very life stems from the fact that it is not a tree, and that the receptacle which grows naturally around it, when it is not artificially conceived, mirrors this semi-lattice structure and the overlap within it. Finally, in the last section, I shall try to explain that it is the process of thought itself which works in a tree-like way, so that whenever a city is "thought out,"

instead of "grown," it tends to get a tree-like structure.

III. ARTIFICIAL CITIES WHICH ARE TREES

Let us now test the theory that artificial cities are trees, while old and natural cities are semi-lattices. I shall first describe twenty modern conceptions of the city, showing that each one is essentially a tree.

It will perhaps be useful, while we look at these plans, to have a little ditty in our minds.

Big fleas have little fleas
Upon their back to bite em,
And little fleas have lesser fleas,
And so ad infinitum.

This rhyme expresses perfectly and succinctly the structural principle of the tree.

1. Cluster city. A. and P. Smithson.

The basic unit is the house. Groups of houses form streets. Groups of streets form clusters. And groups of clusters form a city. The organization is a tree.

Figure 1

2. Greenbelt, Maryland. Clarence Stein.

The city is broken down into superblocks. Each superblock contains schools, parks, and a number of subsidiary groups of houses built round parking lots. The organization is a tree.

Figure 2

3. Greater London plan (1943). Abercrombie and Forshew.

The drawing shows the structure conceived by Abercrombie for London. It is made of a large number of "communities" each sharply separated from all adjacent communities. On page 28 Abercrombie writes: "The

proposal is to emphasize the identity of the existing communities, to increase their degree of segregation, and where necessary to reorganise them as separate and definite entities." And on page 26, "The communities themselves consist of a series of sub-units, generally with their own shops and schools, corresponding to neighbourhood units." The city is conceived as a tree with two principal levels. The communities are the larger units of the structure; the smaller subunits are neighbourhoods. There are no overlapping units. The structure is a tree.

Figure 3

4. Middlesborough redevelopment plan (194). Ruth Glass.

Ruth Glass's plan, which I have not illustrated, recommends that Middlesborough, a town of 200,000 people be broken into 29 entirely separate neighborhoods; in other words, that it be a tree.

Figure 4

5. CIAM 8: The heart of the city (1952), pp. 164-165. J. Tyrwhitt, J. L. Sert, E. N. Rogers; "The hierarchy of urban cores."

The authors argue that since "the core is the expression of general factors of human nature," the city should be provided with a hierarchy of cores, each appropriate to the size area served, each dependent on its superior cores, each with its own dependent subcores. This is a tree of cores.

We find this principle embodied in two plans, both actually published before the CIAM meeting.

Figure 5

6. The MARS plan for London (1943).

The entire city is dependent on the striplike commercial industrial core at the center. Hung from this center are lesser striplike

communities, each dependent on its own core.

Figure 6

7. Chandigher (1951). Le Corbusier.

The whole city is served by a commercial center in the middle, linked to the administrative center at the head. Two subsidiary elongated, commercial cores are strung out along the major arterial roads, running north-south. Subsidiary to these are further administrative, community and commercial centers, one for each of the city's 20 sectors.

Figure 7

8. Another document with a widespread effect on three thinking, was the Athens Charter, CIAM 1929, which effectively proposed that the city be given tree organization as in the following figure.

Figure 8

9. Cite Industrielle. Tony Garnier.

This precursor of the Athens Charter, proposed an even simpler basic tree structure.

Figure 9

10. Communitas. Percival and Paul Goodman.

Goodman's Communitas is explicitly organized as a tree: it is first divided into four concentric major zones, the innermost being a commercial center, the next a university, the third residential and medical, and fourth open country. Each of these is further subdivided: the commercial center is represented as a great cylindrical skyscraper, containing five layers, airport, administration, light manufacture, shopping and amusement, and at the bottom railroads, buses and mechanical services. The university is divided into eight sectors comprising, natural history, zoos and aquariums, planetarium, science, laboratories, plastic arts, music and drama. The third concentric ring is divided

into neighborhoods of 4,000 people each, not consisting of individual houses, but of apartment blocks, each of these containing further individual dwelling units. Finally, the open country is divided into three segments, forest preserves, agriculture, and vacation-lands. The over-all organization is a tree.

Figure 10

11. Ideal City (1947). Jean-Claude Mazet.

An extreme example of the Athens Charter ideas is to be found in the city by Jean-Claude Mazet, in which all sports, games, and gardens are relegated to E and F, entirely outside the pyramid C, which houses the city, which is itself totally separated from circulation A, D, J.

Figure 11

12. Mesa City, Paolo Soleri.

The organic shapes of Mesa City may lead a careless glance to believe that it is a richer structure than our more obviously rigid examples. But when we look at it in detail we find precisely the same principle of organization. Take, particularly, the university center. Here we find the center of the city divided into a university, separated from a residential quarter, which is itself divided into a number of villages (actually apartment towers) for 4,000 inhabitants, each again subdivided further and surrounded by groups of still smaller dwelling units.

Figure 12

13. The seven V's. Le Corbusier.

An extension of the idea of separating traffic from pedestrians, this defines 7 speeds, V-1 for ultra high speed circulation and V-7 for walking. The assumption is that each single artery of any one speed, serves a number of arteries of the next slowest speed. This a thought, rather than a plan, but it is a tree.

Figure 13

14. Rivers, harbors, canals, docks. Louis Kahn.

This is a very similar thought, expressing the fact that each larger kind of circulation serves the smaller kinds which branch out from it.

Figure 14

15. Brasilia. Lucio Costa.

Here we have a city which embodies these ideas. The entire form pivots about the central axis; each of the two halves is served by a single main artery. This main artery is in-fed by subsidiary arteries parallel to it; finally these are fed by the roads which surround the superblocks themselves. The structure is a tree.

Figure 15

16. Tokyo plan. Kenzo Tange.

This is a beautiful example. The plan consists of a series of loops stretched across the Tokyo Bay. There are four major loops. Each of these major loops contains three medium loops. In the second major loop, one medium loop is the railway station, another is the port. Otherwise each medium loop contains three minor loops, which are residential neighborhoods, except in the third major loop where one contains government offices, and another contains industrial offices.

Figure 16

17. Regional science (the theory of market regions). August Losch.

The theory is based on the following question: Given an endless spatial distribution of settlements (which he assumes to be on a hexagonal grid), what is the relation between the producers of some product, and the market supplied by that producer? By weighing the cost of distribution against the demand, he shows that each product has an optimum radius of distribution; he then argues that the producers of such goods will distribute themselves as nearly as possible to make their market circles

a packing of optimum radius circles. Losch observes, naturally, that even within his premises, since different products have a different maximum marketing radius, the market regions for different products will overlap. You will not be surprised to learn that at this point, however, instead of accepting the overlap, he starts to discuss the beauty and simplicity of the tree structure, in which the only different radii that are allowed are those that bear a tree-like relation to one another. Thus in the two examples shown, each region contains four (seven) regions of the next smaller size.

Figure 17

18. Since I am about to attack the foundations of all of these plans, and have myself been guilty of the same mistake, I cannot leave myself off the roster of criminals. Only a year ago I made an analysis of an Indian village, with the intention of finding the most proper physical components out of which the village should be built. The analysis I made, picture below, is again a tree—it is a system of four major components, each subdivided into further, smaller components. The tree takes no account of the overlap between components which would actually be required to encompass the village functions.

Figure 18

19. The most beautiful example of all, I have kept till last, because it symbolizes the problem perfectly. It appears in Hilbersheimer's book called The Nature of Cities (p.148). He describes the fact that certain Roman towns had their origin as military camps—and then shows a picture of a modern military encampment as a kind of archetypal form for the city. It is not possible to have a structure which is a clearer tree. The regiment is divided into battalions; the battalions into companies; the companies into platoons; the platoons into sections.

Each of these units has its commanding officer; in the plan, the tent of the commanding officer marks the unit.

The symbol is apt, for, of course, the organization of the army was created precisely in order to create discipline and rigidity. When a city is endowed with a tree structure, this is what happens to the city and its people.

Figure 19

20. Finally, here is Hilbersheimer's own scheme for the commercial area of a city based on the army camp archetype.

Figure 20

It is clear, just from the way I have described them, that each of these structures is a tree. And we know from the criterion for a tree just what this means, formally. But so that we get a really clear understanding of what it means, and shall better see its implications, let us define it again.

Whenever we have a tree structure, it means that within this structure no piece of any unit is ever connected to other units, except through the medium of that unit as a whole.

The enormity of this restriction is difficult to grasp. It is a little as though the members of a family were not free to make any friends outside the family, except when the family as a whole made a friendship.

In a strictly grammatical sentence too, the structure is tree-like. The noun only works within its noun phrase, the object as part of the predicate. The object, because it is part of the predicate, can relate to the subject only through the medium of the whole predicate. It is only in poems or dreams that we throw away this simple-minded conception of structure, and allow many things to relate to each other in many different ways,

not always through the medium of the same superordinate structures. Is the city like a poem, or is it to be like a dull statement?

It must be emphasized, I think, lest the orderly mind shrink in horror form anything that is not clearly articulated and categorized in tree form, that the idea of overlap, ambiguity, multiplicity of aspect, and the semi-lattice, are not less orderly than the rigid tree, but more so. They represent a thicker, tougher, more subtle and more complex view of structure.

For complexity of structure the three is comparable to the compulsive desire for neatness and order that insists the condlesticks on a mantle-piece be perfectly straight and perfectly symmetrical about the center. The semi-lattice by comparison, is the structure of a complex fabric, it is the structure of living things, of great paintings and symphonies.

Let us now look at the ways in which the natural city, when unconstrained by artificial conceptions, shows itself to be a semi-lattice.

IV. THE LIVING CITY IS AND NEEDS TO BE A SEMI-LATTICE

Each unit in each tree that I have described, is the fixed invariant residue of some system in the living city. A house is the residue of the interactions between the members of a family, their emotions, and their belongings; a freeway the physically persistent evidence of movement and commercial exchange.

However, in every city there are thousands, even millions, of times as many more systems at work, whose physical residue does not appear as a unit in these tree structures. In the worst cases, the units which do appear fail to correspond at any living reality; and the real systems, whose existence actually makes the city live, have been provided with no physical receptacle.

Neither the Smithson plan nor the Stein plan, correspond to social realities. The physical layout of the plans, and the way they function, suggests a hierarchy of stronger and stronger closed groups, ranging from the whole city down to the family, each formed by associational ties of different strength. In a traditional society, if we ask a man to name his best friends, and then ask each of these in turn to name their best friends, they will all name each other—so that they form a closed group— and a village is made of a number of separate closed groups of this kind. But todays social structure is utterly different. If we ask a man to name his friends, and then ask them in turn to name their friends, they will all name different people, very likely unknown to the first person, and these people would again name others, and so on outwards. There are virtually no closed groups of people in modern society: the reality of todays social structure is thick with overlap—the systems of friends and acquaintances form a semi-lattice, not a tree.

In the natural city, even the house on a long street (not in some little cluster) is a more accurate acknowledgment of the fact that your friends live not next door, but far away, and can only be reached by bus or automobile. In this respect Manhattan has more overlap in it than Greenbelt. And though one can argue that in Greenbelt too, friends are only minutes away by car, one must ask then: "Since certain groups <u>have</u> been emphasized by the physical units of the physical structure, why are just these the most irrelevant ones?"

Another aspect of the city's social structure which these trees fail to mirror properly, is illustrated by Ruth Glass's own study. After picking her 29 Middlesborough neighborhoods "by eye," that is, by determining where the sharpest discontinuities of building-type, income, job-type, occur, she asks herself the question: "If we examine some of the social systems which actually exist for the people in such a neighborhood, do the physical units defined by these various social systems all define the same spatial neighborhood?" Her own answer to this question is, <u>no</u>.

Each of the social systems she examines is a nodal system: it is made of some sort of central node, plus the people who use this center. Specifically she takes elementary schools, secondary schools, youth clubs, adult clubs, post offices, greengrocers, and grocers selling sugar. Each of these centers draws its users from a certain spatial area or spatial unit. This spatial unit is the physical residue of the social system as a whole, and, therefore, a unit in the terms of this paper. The units corresponding to different kinds of center, for a single neighborhood, Waterloo Road, are shown below.

Figure

The hard outline is the boundary of the so-called neighborhood itself. The gray circle stands for the youth club, and the small solid rings stand

for areas where its members live. The ringed spot is the adult club, and the homes of its members form the unit marked by dashed boundaries. The square is the post office, and the dotted line marks the unit which contains its users. The secondary school is marked by the spot with a white triangle in it; together with its pupils, it forms the system marked by the dot-dashed line.

As you can see at once, the different units do not coincide. Yet neither are they disjoint. They overlap.

We cannot get an adequate picture of what Middlesborough is, or of what it ought to be, in terms of 29 large and conveniently integral chunks called neighborhoods. When we describe the city in terms of neighborhoods, we implicitly assume that the smaller elements within any one of these neighborhoods belong together so tightly that they only ever interact with elements in other neighborhoods through the medium of the neighborhood they themselves belong to. Ruth Glass herself shows clearly that this is not the case.

Illustrated below are two pictures of the Waterloo "neighborhood." For the sake of argument I have broken it into 23 pieces. Figure 22a shows how these pieces stick together in fact, and Figure 22b shows how the redevelopment plan pretends they stick together.

There is nothing in the nature of the various cente which says why their catchment areas should be the same. Their natures are different: therefore the units they define are different. The natural city of Middlesborough was faithful to the semi-lattice structure they have. Only in the artificial tree conception of the city are their natural, proper, and necessary overlaps destroyed.

The very same oversimplification as Ruth Glass's appears in Lösch's theory of market regions.

The essence of the theory is that under the most efficient working of the economy, each buyer would end up with just one supplier for any particular product—the one who happened to be at the center of the market region the buyer lives in. Yet this monopolistic situation is exactly what neither buyer nor sellers in the city allow. Do you always go to the same supermarket, the same hardware store, the same clothing store? You do not. And in fact, even when you settle into the comfortable habit of going to one store most of the time, another store will often try and get your custom by providing better service, and you welcome it. It is the fact that the second store may try to do this, that makes the market free. The real market regions in a natural city, if we define a market region as the catchment area of a particular store, overlap to a fantastic extent. The situation where there is no overlap represents exactly all that we do not like about totalitarianism, where each buyer is forced to buy from the particular store assigned him. A tree of markets can only be the basis for a totalitarian nonopolistic economy—the basic idea of democracy, to offer and foster choice, demands a semi-lattice of market regions. Yet the neighborhood shopping center, the hierarchy of urban cores, the plans of Brazilia, Chandighar, and the British New Towns, all created for supposedly free societies, all offer their inhabitants a tree of markets.

Figure

In these examples the tree structure is wrong, because the systems it is supposed to contain simply are not trees—it is their nature to overlap. But the tree is not always actually wrong—it is more often merely crude: yet so crude that it actually destroys all the detail which make life's fabric.

Take the separation of pedestrians from moving vehicles, a tree concept proposed by Le Corbusier, and Louis Kahn, and many others. At a very crude

level of thought this is obviously a "good idea." It is dangerous to have 60-mile-an-hour cars in contact with little children toddling. But it is not always a good idea: there are times when the ecology of a situation actually demands the opposite. Imagine yourself coming out of a Fifth Avenue store; you have been shopping all afternoon; your arms are full of parcels; you need a drink; your wife is limping. Thank God for taxis.

Yet the urban taxi can function only because pedestrians and vehicles are not strictly separated. The prowling taxi needs a fast stream of traffic so that it can cover a large enough area to be sure of finding a passenger. The pedestrian needs to be able to hail the taxi from any point in the pedestrian world, and to be able to get out at any part of the pedestrian world he wants to go to. The system which contains the taxicabs needs to overlap both the fast vehicular traffic system, and the system of pedestrian circulation. In Manhattan pedestrians and vehicles do share certain parts of the city: and the necessary overlap is guaranteed.

Another example of this overlap in a natural city, arises for a different reason. The sidewalks on the Champs Elysées have parked cars on them. A car moving at three miles an hour among so many people can harm no one: and the result is that instead of parking in an isolated parking garage, you park right where you want to be: as you leave your car you plunge into the crowd.

In both these cases the semi-lattice which occurs naturally, and is required, contains places which the car and vehicle both use: the unit which the cars are in, and the unit which people are in, overlap.

Figure

Another feature of the Athens Charter tree, realized in extreme form in Mazet's city, is the separation of recreation from everything else. This

has crystallized, in our real cities, in the form of playgrounds. The playground, asphalted, fenced in, is nothing but a pictorial acknowledgment of the fact that "play" exists as an isolated concept in our minds. It has nothing to do with the life of play itself. No self-respecting child will even play in a playground.

Play itself, the play that children practice, goes on somewhere different everyday. One day it may be indoors, another day in a friendly gas station, another day down by the river, another day in a derelict building, another day on a construction site which has been abandoned for the weekend. Each of these play activities and the objects it requires, form a systen. It is not true that these systems exist in isolation, cut off from the other sysrems in the city. The different systems overlap one another, and they overlap many other systems besides: the units, the physical places recognized as play places, must do the same.

In a natural city this is what happens. Play takes place in a thousand places--it fills the interstices of adult life. As they play, children becoms full of their surroundings. How can a child become filled with his surroundings in a fenced enclosure? He cannot. In a semi-lattice he can; in a tree he can't.

A similar kind of mistake occurs in trees like that of Goodman's Commitas, or Soleri's Mesa City, which separate the university from the rest of the city. Again, this has actually been realized in the common American form of the isolated campus.

Figure

What is the reason for drawing a line in the city, so that everything within the boundary is university, and everything outside is non-university. It is conceptually clear. But does it correspond to the realities of university life. Certainly it is not the structure which occurs in non-artificial

university cities.

Take Cambridge University, for instance. At certain points Trinity street is physically almost indistinguishable from Trinity College. One pedestrian crossover in the street is literally part of the college; the buildings on the street, though they contain stores and coffee shops and banks at ground level, contain undergraduates rooms in their upper stories; in many cases the actual fabric of the street buildings melts into the fabric of the old college buildings so that one cannot be altered without the other.

There will always be many systems of activity where university life and city life overlap: pub-crawling, coffee-drinking, the movies, walking from place to place; in some cases whole departments may be actively involved in the life of the city's inhabitants (the hospital-cum-medical school is an example). In Cambridge, a natural city where university and city have grown together gradually, the physical units overlap because they are the physical residues of city systems and university systems which overlap.

Let us look next at the hierarchy of urban cores, realized in Brazilia, Chandighar, the MARS plan for London, and, most recently, in the Manhattan Lincoln Center, where various performing arts, all serving the population of greater New York, have been gathered together to form just one core.

Does a concert hall ask to be next to an Opera House. Can the two feed on one another. Will anybody ever visit them both, gluttonously, in a single evening, or even buy tickets from one after going to a concert in the other. In Vienna, London, Paris, where you will, each of the performing arts has found its own places, which, just because they are not mixed randomly, have created their own magic, each in its own familiar section of

the city. In Manhattan itself Carnegie Hall and the Metropolitan Opera House were not built side by side. Each found its own place, and now creates its own atmosphere. The influence of each overlaps the parts of the city which have been made unique to it.

The only reason that these functions have all been brought together in the Lincoln Center, is that the concept of performing art links them to one another.

Figure

Butthis tree, and the idea of a single hierarchy of urban cores which is its parent, do not illuminate the relations between art and city life--they are merely born of the mania every simple-minded person has, for putting things with the same name into the same basket.

The total separation of work from housing, started by Tony Garnier in his industrial city, then incorporated in the 1929 Athens Charter, is now found in every artificial city and accepted everywhere where zoning is enforced. In this respect is this a sound principle? It is easy to see how bad conditions at the beginning of the century prompted planners to try and get the dirty factories out of residential areas. But the separation misses a variety of systems which require, for their sustenance, little parts of both.

Jane Jacobs describes the growth of back-yard industries in Brooklyn. A man who wants to start a small business needs space, which he is very likely to have in his own back yard, and also needs to establish connections with larger going enterprises, and with their customers. This means that the system of back-yard industry needs to belong both to the residential zone, and to the industrial zone: these zones need to overlap. In Brooklyn they do. In a city which is a tree, they can't.

Figure

Let us consider another facet of city life: its industrial and mercantile activities. In Hilbersheimer's commercial sector of a city, figure 20, these activities are placed in a container whose physical units have a tree structure. But listen to Durkheim, talking about the overlap of social groupings caused by the division of labor in society (p. 125):

"Good examples are to be found in the complexity of groupings defined by contracts: contracts between the owner of a ship and his creditors, between carrier and skipper, between the owner and captain plus crew, between the grantor of a charter and the charterer, between the insurer and insured." Each contract defines a system of the social life—evidently, the fact that a given individual may enter into several different kinds of contract simultaneously indicates that the systems overlap.

If they are to work properly, the overlap of these social systems must be mirrored by the overlap of physical units which contain them. The physical structure too, must be a semi-lattice, not a tree.

Let us consider finally, the subdivision of the city into isolated communities. As we have seen, in the Abercrombie plan for London, this is itself a tree structure. The individual community in a greater city, has no reality as a functioning unit. In London, as in any great city, almost no one manages to find work which suits him, near his home. People in one community work in a factory which is very likely in another community. There are, therefore, many hundreds of thousands of worker-workplace systems, each consisting of a man plus the fadtory he works in, which cut across the boundaries defined by Abercrombie's tree. The existence of these units, and their overlapping nature, indicates that the living systems of London form a semi-lattice—only in the planner's mind has it become a tree.

The fact that we have so far failed to give this any physical expression, has a vital consequence. As things are, whenever the worker and his workplace

belong to separately administered municipalities, the community which contains the workplace collects huge taxes, and has relatively little to spend the tax money on, while the community where the worker lives, if it is mainly residential, collects only little in the way of taxes, and yet has great additional burdens on its purse, in the shape of schools, hospitals, etc. Clearly, to resolve this inequity, the worker-workplace systems must be anchored in physically recognizable units of the city, which can then be taxed.

It might be argued, that even though the individual communities of a great city have no functional significance in the lives of their inhabitants, they are still the most convenient administrative units, and should, therefore, be left in their present tree organization.

However, in the political complexity of a modern city, even this is suspect.

Edward Banfield, in a recent book called Political Influence, gives a detailed account of the patterns of influence and control that have actually led to decisions in Chicago. He shows that although the lines of administrative and executive control have a formal structure which is a tree, these formal chains of influence and authority are entirely overshadowed by the ad hoc lines of control which arise naturally as each new city problem presents itself. These ad hoc lines depend on who is interested in the matter, who has what at stake, who has what favors to trade with whom. This second structure, which is informal, working within the framework of the first, is what really controls public action. It varies from week to week, even from hour to hour, as one problem replaces another. Nobody's sphere of influence is entirely under the control of any one superior; each person is under different influences as the problems change. Although the organization chart in the mayor's office is a tree, the actual control and exercise of authority is semi-lattice-like.

Let me finally give an example from the Indian village. Unit A, which has chiefly to do with the activities of cattle, their grazing, shelter, milking, dung-collection, etc., has been separated entirely from Unit B which encompasses mainly agricultural activities. Specifically A-1, which is the unit that takes care of cattle grazing and milking, is thoroughly separated from Unit B-3 which comprises terraced strips of alternating agricultural land, paths, and irrigation channels.

From most points of view this separation is efficient. Planting and irrigation are separate from grazing and milking. But situations can arise which the separation fails to accommodate. A farmer uses a bullock cart to bring vegetables from his field to his house, and wants to leave the bullock grazing at the fields edge while he loads the cart. Another farmer has no children who can take his cattle to graze, and doesn't wish to entrust his animals to anyone else's care, wants to take them to the field's edge for the whole day, where they can graze under his supervision while he works in the field. To accommodate these situations in a natural village there are strips of grass along the field's edges.

It can be argued, of course, that these strips represent an inefficient use of land, and that the grazing grounds should be consoliated as in A-1. But such efficiency fails to accommodate real interactions.

The village requires a further unit, besides A-1 and B-3--as simple as the grass sown between fields, perhaps, but less wasteful--which would extend both A-1 and B-3, to overlap one another. This new unit makes the village a semi-lattice, as the natural village is.

Figure

The tree, though so neat and beautiful as a mental device, though it offers such a simple and clear way of dividing a complex entity into units,

fails to describe correctly the actual structure of naturally occurring cities, and fails to describe the structure of the cities which we need.

We have found indeed, that the only cities which conform even approximately to the tree pattern, are those artificial cities and parts of cities whose creation has actually been derived from mental constructs which are trees.

V. THE TREE IS A DEEP AND PERSISTENT MENTAL HABIT

Now, why is it that so many designers have conceived cities as trees, when the natural structure is in every case a semi-lattice? Have they done so deliberately, in the belief that a tree structure will serve the people of the city better? Or have they done it because they cannot help it, because they are trapped by a mental habit, perhaps even trapped by the way the mind works, because they cannot encompass the complexity of a semi-lattice in any convenient mental form, because the mind has an overwhelming predisposition to see trees wherever it looks and cannot escape the tree conception?

I shall try to convince you that it is for this second reason that trees are being proposed and built as cities--that it is because designers, limited as they must be by the capacity of the mind to form intuitively accessible structures, cannot achieve the complexity of the semi-lattice in a single mental act.

Let me begin with an example.

Suppose I ask you to remember the following four objects: an orange, a watermelon, a football, and a tennis ball. How will you keep them in your mind, in your mind's eye? However you do it, you will do it by grouping them. Some of you will take the two fruits together, the orange and the watermelon, and the two sports balls together, the football and the tennis ball. Those of you who tend to think in terms of physical shape, may group them differently, taking the two small spheres together, the orange and the tennis ball, and the two larger and more egg-shaped objects, the watermelon and the football. Some of you will be aware of both.

Let us make a diagram of these groupings.

Figure

Either grouping taken by itself is a tree structure. The two together are a semi-lattice. Now let us try and visualize these groupings in the mind's eye. I think you will find that you cannot visualize all four classes simultaneously—because they overlap. You can visualize one, and then the other, and you can alternate between the two extremely fast—so fast that you may deceive yourself into thinking you can visualize them all together. But in truth, you cannot conveive all four at once in a single mental act. You cannot bring the semi-lattice structure into a visualizable form for a single mental act; in a single mental act you can only visualize a tree.

This is the problem we face as designers. While we are not, perhaps, necessarily occupied with the problem of total visualization in a single mental act, the principal is still the same. The tree is accessible mentally, and easy to deal with. The semi-lattice is hard to keep before the mind's eye, and, therefore, hard to deal with.

It is known today, that grouping and categorization are among the most primitive psychological processes. Modern psychology treats thought as a process of fitting new situations into existing slots and pidgeon holes in the mind. Just as you cannot put a physical thing into more than one physical pidgeon hole at once, so, by analogy, the processes of thought prevent you from putting a mental construct into more than one mental category at once. Study of the origin of these processes, suggests that they stem essentially from the organism's need to reduce the complexity of its environment by establishing equivalences between the different events which it encounters.

In a field full of many mushrooms, both edible and poisonous, a man's best way of surviving will be to learn which mushrooms can be treated as equivalent, so that the same response can appropriately be given to all

equivalent ones. He does this by creating visual conceptions and verbal names which specify types of mushrooms: since the mushrooms of any type are equivalent, a man who finds a new mushroom can decide whether it is edible or not, very easily, by identifying its type.

Now it is a mathematical fact that an equivalence relation on a universe of things, partitions that universe into nonoverlapping classes. And it is only this fact which ensures survival. If the classes overlapped, so that there were certain mushrooms which belonged both to the poisonous class, and to the class of edible ones, sooner or later there would be an accident. The organism cannot afford ambiguity in the classifications it creates.

It is for this reason, because the mind's first function is to reduce the ambiguity and overlap in a confusing situation, and because, to this end, it is endowed with a basic intolerance for ambiguity, that structures like the city, which do require overlapping sets within them, are nevertheless persistently conceived as trees.

The same rigidity dogs even the perception of physical patterns. In experiments by Huggins and myself at Harvard, we showed people patterns whose internal units overlapped, and found that they almost always invented a way of seeing the pattern as a tree-even when the semi-lattice view of the pattern would have helped them perform the experimental task in hand.

The most startling proof that people tend to conceive even physical patterns as trees, is found in some experiments of Sir Frederic Bartlett's. He showed people a pattern, for about 1/4 second, and then asked them to draw what they had seen. Many people, unable to grasp the full complexity of the pattern they had seen, simplified the patterns by cutting out the overlap. Below on the right are two "fairly typical" redrawn versions of the original figure shown on the left. In the redrawn versions the

circles are separated from the rest; the overlap between triangles and circles disappears.

Figure

In a similar experiment by Rupp, people were asked to copy a simple honeycomb pattern. Again, the redrawn versions eliminate the overlap. The common boundary of the adjacent hexagons, which is the zone of overlap between adjacent units, has disappeared. What starts, perceptually, as a semi-lattice, is redrawn as a tree.

Figure

These experiments suggest strongly that people have an underlying tendency, when faced by a complex organization, to reorganize it mentally in terms of nonoverlapping units. The complexity of the semi-lattice is replaced by the simpler and more graspable tree form.

To see how our tendency to conceive things as trees extends even to everyday objects, I ask you to imagine a simple kitchen knife. We speak of the knife as being made of two pieces, blade and handle. In how many people's conception of a knife, does the handle overlap the blade. Yet this is what happens, and indeed this is why the knife is one object and not two. There are three units, the bone part of the handle, the steel part of the handle, and the blade. The steel part of the handle is physically continuous with the steel blade, and these two form a larger unit. The handle as a whole is made of the bone part together with the steel shaft, and it forms a larger unit. These two larger units overlap. Diagramatically the knife is:

Figure

You are no doubt wondering, by now, what does a city look like which is a semi-lattice, but not a tree. I must confess that I cannot yet show

you plans or sketches. It is not enough merely to make a demonstration of overlap—the overlap kust be the right overlap. This is doubly important, because it is so tempting to make plans in which overlap occurs for its own sake. This is essentially what the high density "life-filled" city plans of recent years do. But overlap alone does not give structure—it can also give chaos. A garbage can is full of overlap. To have structure, you must have the right overlap, and this is for us almost certainly different from the old overlap which we observe in historic cities. As the relationships between functions change, so the systems which need to overlap in order to receive these relationships must also change— the re-creation of old kinds of overlap will be inappropriate, and chaotic instead of structured.

The work of trying to understand just what overlap the modern city requires, and trying to put this required overlap into physical and plastic terms, is still going on. Until the work is complete, there is no point in presenting facile sketches of ill-thought-out structure.

However, I can perhaps make the physical consequences of overlap more comprehensible by means of an image.

The painting illustrated is a recent work by Simon Nicholson.

The fascination of this painting lies in the fact that although constructed of rather few simple triangular elements, these elements unite in many different ways to form the larger units of the painting: in such a way indeed, that if we make a complete inventory of the perceived units in the painting, we find that each triangle enters into four or five completely different kinds of unit, none contained in the others, yet all overlapping in that triangle.

Thus, if we number the triangles and pick out the sets of triangles which appear as strong visual units, we get the semi-lattice shown below.

Figure

3 and 5 form a unit because they work together as a rectangle, 2 and 4 because they form a parallelogram, 5 and 6 because they are both dark and pointing the same way, 6 and 7 because one is the ghost of the other shifted sideways, 4 and 7 because they are symmetrical with one another, 4 and 6 because they form another rectangle, 4 and 5 because they form a sort of Z, 2 and 3 because they form a rather thinner kind of Z, 1 and 7 because they are at opposite corners, 1 and 2 because they are a rectangle, 3 and 4 because they point the same way like 5 and 6, and form a sort of off-center reflection of 56, 3 and 6 because they enclose 4 and 5, 1 and 5 because they enclose 2, 3, and 4. I have only listed the units of two triangles. The larger units are even more complex. The white is more complex still, and is not even included in the diagram, because it is harder to be sure of its elementary pieces.

The painting is significant, not so much because it has overlap in it (many paintings have overlap in them), but rather because this painting has nothing else in it except overlap. It is only the fact of the overlap, and the resulting multiplicity of aspects which the forms present, that makes the painting fascinating. It seems almost as though the painter had made an explicit attempt, as I have done, to single out overlap as a vital generator of structure.

By contrast, an image whose perceptual structure is purely tree-like, is shown below. Here, each little square belongs only to one larger square of four little squares, and these larger squares themselves belong only to the superordinate squares of sixteen squares, and so on.

Figure

All the artificial cities I have described have the structure of this second image, rather than the structure of the Nicholson painting. Yet

it is the painting, and other images like it, which must be our vehicles for thought. And when we wish to be precise, the semi-lattice, being part of a large branch of modern mathematics, is a powerful way of exploring the structure of these images. It is the semi-lattice we must look for, not the tree.

When we think in terms of trees we are trading the humanity and richness of the living city, for a conceptual simplicity which benefits only designers, planners, administrators and developers. Every time a piece of a city is torn out, and a tree made to replace the semi-lattice that was there before, the city takes a further step toward dissociation.

In any organized object, extreme compartmentalization and the dissociation of internal elements are the first signs of coming destruction. In a society, dissociation is anarchy. In a person, dissociation is the mark of schizophrenia and impending suicide. An ominous example of city-wide dissociation, is the separation of retired people from the rest of urban life, caused by the growth of desert cities for the old like Sun City, Arizona. This separation is only possible under the influence of tree-like thought.

Figure

It not only takes from the young, the company of those who have lived long, but worse, it causes the same rift inside each individual life. As you yourself will pass into Sun City, and into old age, your ties with your own past will be unacknowledged, lost, and, therefore, broken. Your youth will no longer be alive in your old age--the two will be dissociated, your own life will be cut in two.

For the human mind, the tree is the easiest vehicle for complex thoughts. But the city is not, cannot, and must not be a tree. The city